SuperCDMS SNOLAB Solar axion searches

SNOLAB users meeting
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SuperCDMS SNOLAB technology

Measuring electron and nuclear recoils (ER/NR) in Silicon and Germanium crystals.

ER $\longrightarrow$ e/h pairs

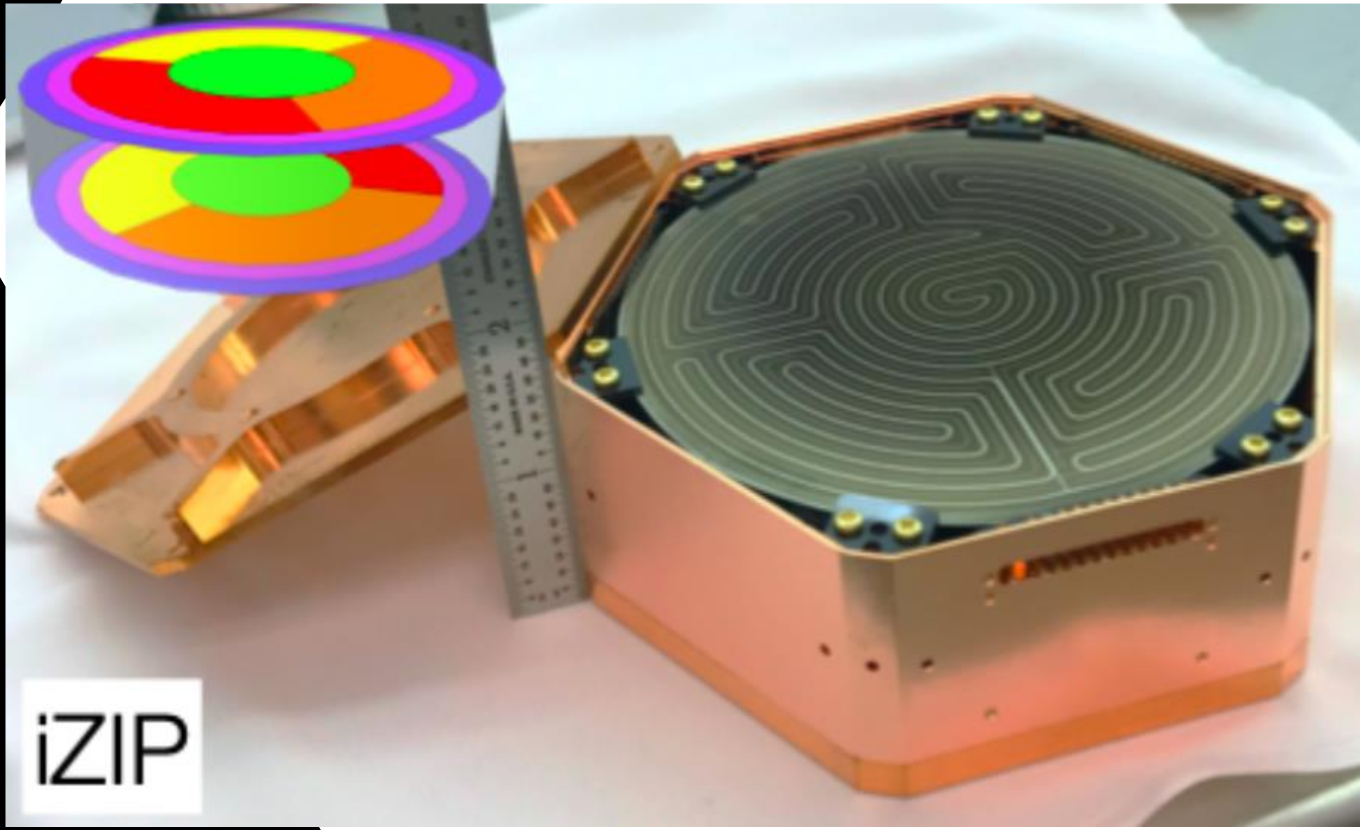
NR $\longrightarrow$ e/h pairs + phonons

iZIP: Interleaved Z-dependent Ionization and Phonon

- Read out both charge and phonon channels.
- NR and ER discrimination for background rejection.

HV: High Voltage

- Read out only phonon channels.
- Higher energy resolution and lower threshold.

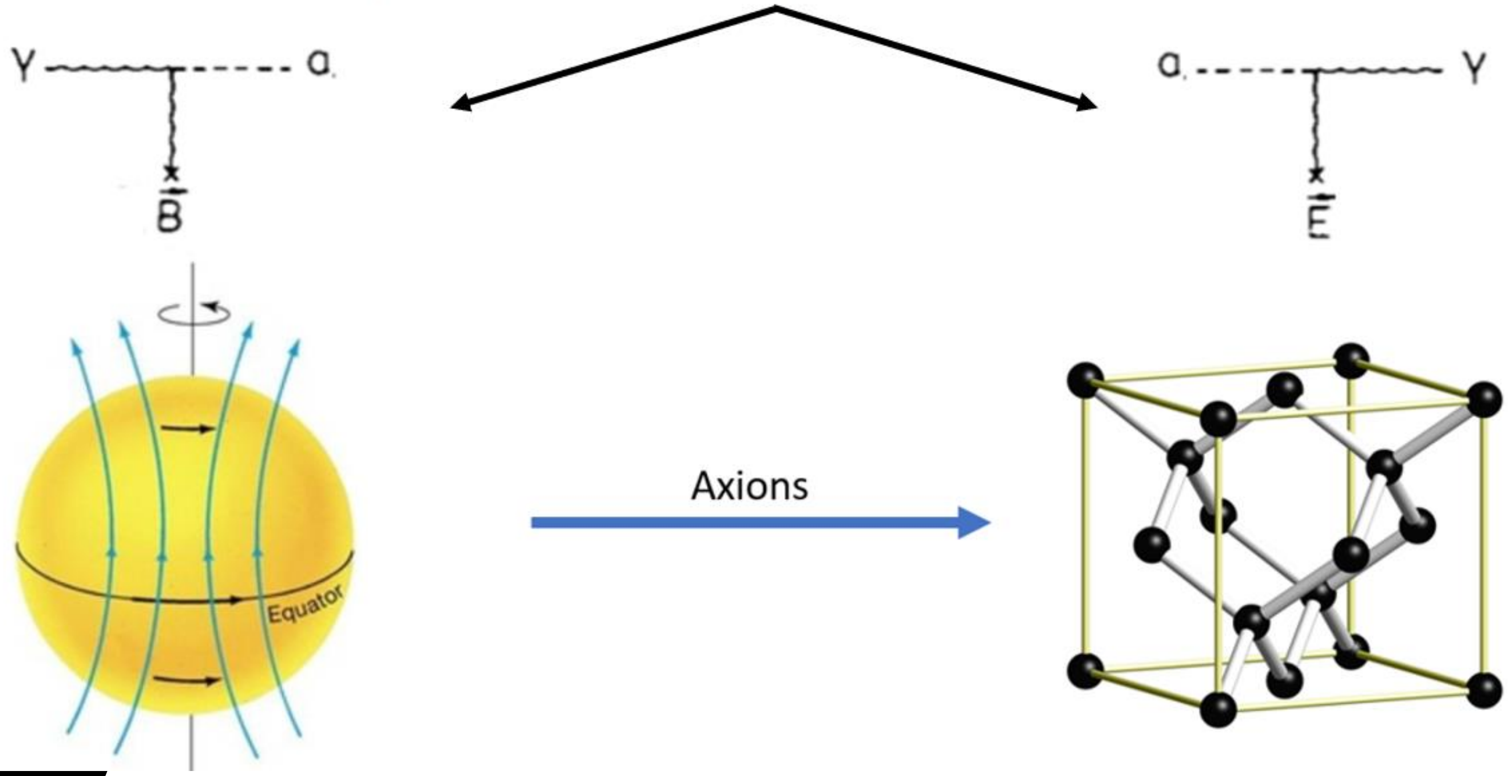


Solar axions detection

- Production: Photons -> Solar axions.
- Conversion: Solar axions -> Photons in the internal electric field of detector crystals.
- Detection: Negligible nuclear recoils, and 1-10 KeV photons producing photoelectric interactions in crystal detectors.

Mixing term:
a = Axion field
$F^{\mu\nu}$ = Electromagnetic density matrix
$g_{a\gamma\gamma}$ = coupling constant

$$\mathcal{L}_{0+a} = -\frac{1}{4\mu_0} F^{\mu\nu} F_{\mu\nu} + \frac{g_{a\gamma\gamma}}{4\mu_0} a F^{\mu\nu} \tilde{F}_{\mu\nu} - A_\mu J_e^\mu + \mathcal{L}_U$$

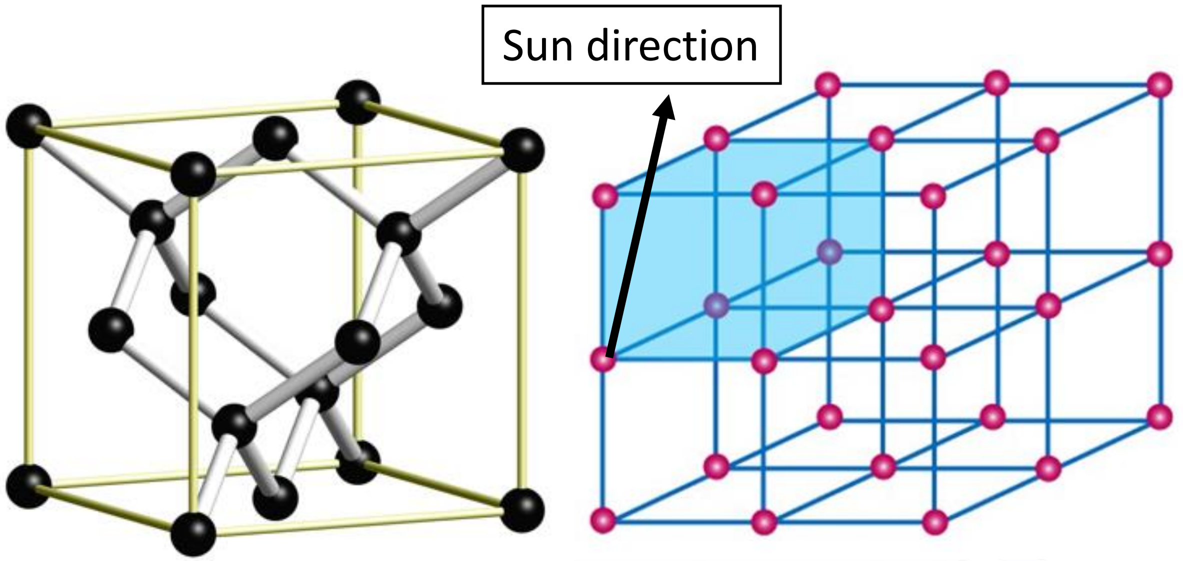


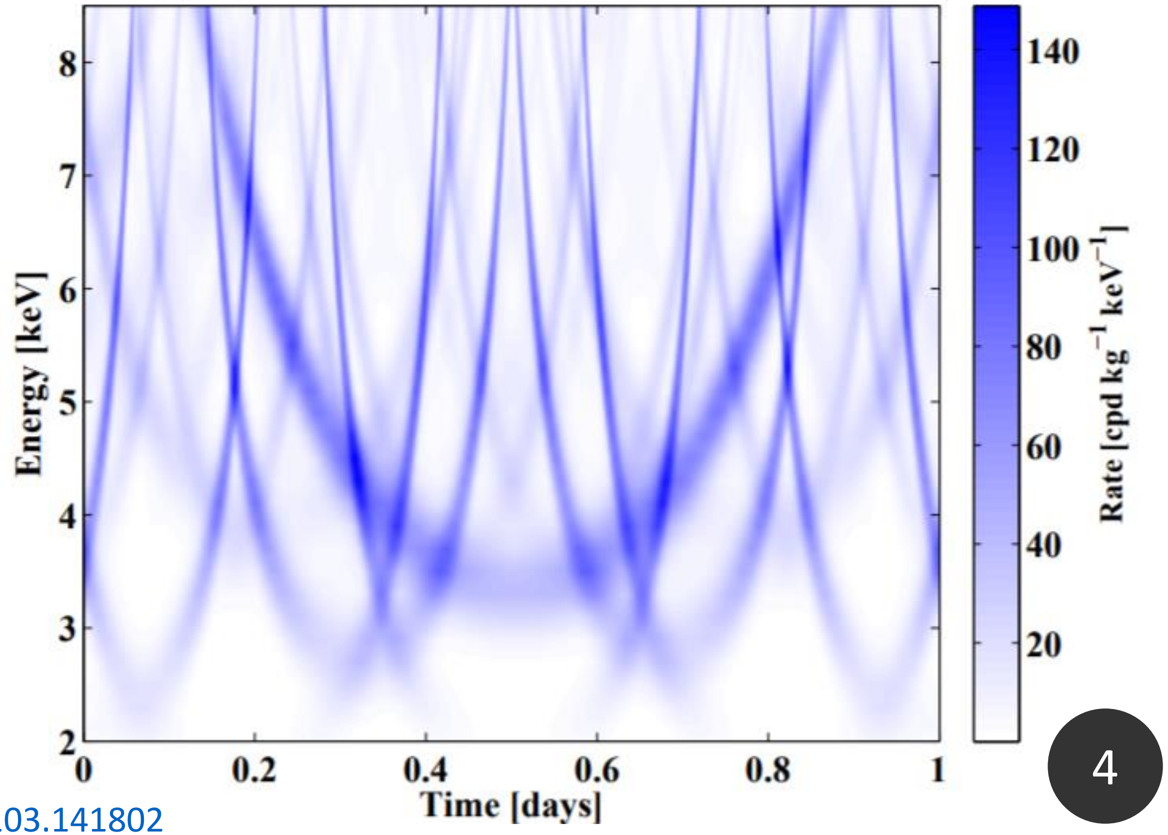
Solar axions signal

Periodic structure of Si/Ge.

-> Periodic electric field.

-> At some incident angles axion-photon conversion rate is higher. So, we expect a time and energy dependent signal.





PhysRevLett.103.141802

Axion-photon conversion rate

1- Modified Maxwell's equations : Axions induce a current density in the crystal.

$$\boldsymbol{j}(\boldsymbol{x},t) = \frac{\eta}{M}\boldsymbol{\mathcal{E}}(\boldsymbol{x}) \times \nabla\phi(\boldsymbol{x},t)$$

$\varepsilon(x)$: Crystal internal electric field
$\emptyset(x,t)$: Axion field

2- Current density generates electric field E.

$$\boldsymbol{E} = \mathbb{G}(\boldsymbol{k},\omega)\cdot\boldsymbol{j}$$

$G(k,w)$: Green's function derived from the Maxwell's equations.

3- Electric field extracts energy from the current density. The energy extraction rate should be proportional to absorption rate of photons in the crystal.

$$\frac{d\mathscr{E}}{dt} = -\int \langle \boldsymbol{j}(\boldsymbol{x},t)\cdot\boldsymbol{E}(\boldsymbol{x},t)\rangle\, d\boldsymbol{x}$$

$$\frac{d\varepsilon}{dt} = \int_0^\infty E_\gamma \frac{dR(E_\gamma)}{dE_\gamma} dE_\gamma$$

Derived from the Maxwell's equations →

$$\bar{\mathbb{G}}_R(\boldsymbol{k},\omega) = -\frac{4\pi^2k^2}{\omega(k+\frac{\omega}{c})}L(k,\omega)\left(\mathbb{U}-\hat{\boldsymbol{k}}\hat{\boldsymbol{k}}\right)$$

$$L(k,\omega) = \frac{1}{\pi}\frac{\frac{\mu(\omega)}{2}}{(k-\frac{\omega}{c})^2+\frac{\mu^2(\omega)}{4}}$$

1/μ = "skin depth"

Absorption length limits

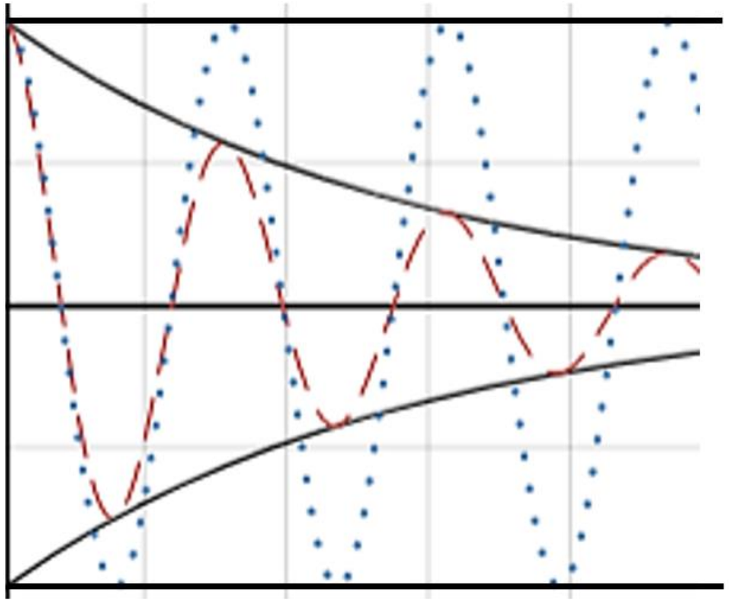
- Conventional method does not consider the electric field attenuation. That corresponds to assuming an infinite absorption length in this work. We analytically checked that.
- We performed numerical calculations with a realistic absorption length.
- The realistic absorption length doesn't have a significant effect on the detection rate of photons.

$$L(k,\omega) = \frac{1}{\pi}\frac{\frac{\mu(\omega)}{2}}{(k-\frac{\omega}{c})^2 + \frac{\mu^2(\omega)}{4}}$$

1/μ = "skin depth"

$$\boldsymbol{E} = \mathbb{G}(\boldsymbol{k},\omega)\cdot\boldsymbol{j}$$

$$|e^{ikz}| = e^{-\mu z}$$



1/μ -> ∞ Conventional method

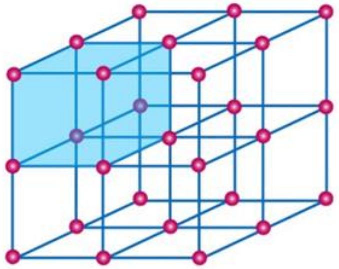
1/μ -> Finite New method

1/μ = "skin depth"

No attenuation ——— 1/μ -> ∞

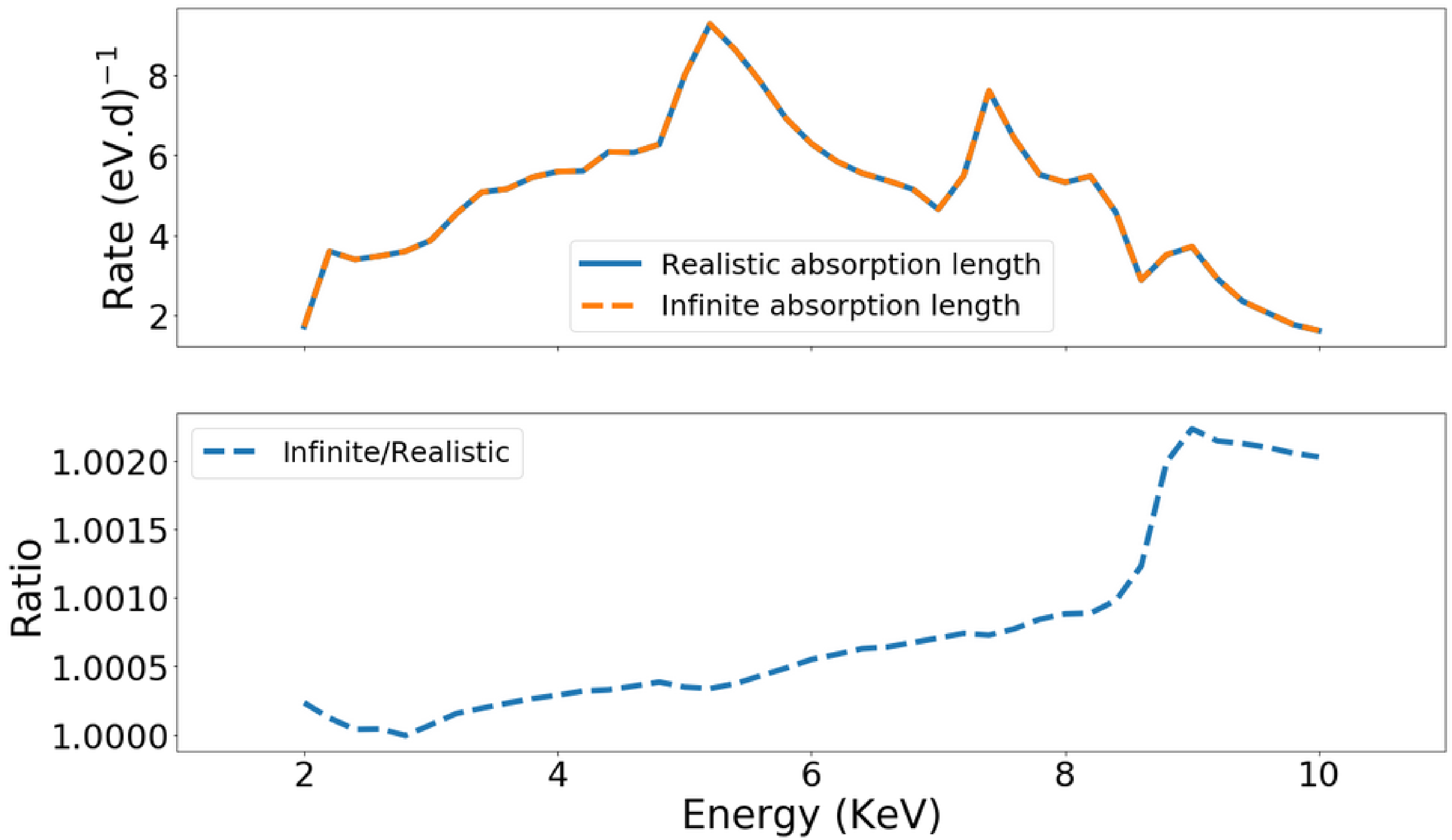
Realistic absorption ——— 1/μ -> $1 - 100\ \mu m$

Si Lattice constant - $0.5\ nm$



Daily averaged event rate example

Expected event rate on 2020/01/01



Conclusion and next step

- Better insight by developing a new method to find the conversion rate of axions and subsequently detection rate of photons in crystal detectors.
- We can safely assume that a realistic absorption length would not have significant effects.
- Preparing an analysis framework to establish limits in axion like parameter space using the SuperCDMS SNOLAB data.

THANKS

Back up slides

Axion-photon conversion rate

There are various techniques to calculate the axion-photon conversion rate in crystals.

$$R(E_1, E_2) = \int_{E_1}^{E_2} dE_{ee} \int_0^{\infty} dE_\gamma \frac{dR}{dE_\gamma}(E_\gamma) \frac{1}{\Delta\sqrt{2\pi}} e^{-\frac{(E_{ee}-E_\gamma)^2}{2\Delta^2}}$$

Production rate

weighting function
(Resolution of detectors)

Writing the differential cross-section for the entire crystal (conventional method.)

$$\frac{d\sigma}{d\Omega} = \frac{g_{a\gamma\gamma}^2}{16\pi^2} F_a^2(\vec{q}) \sin^2(2\theta)$$

$$F_a(\vec{q}) = k^2 \int d^3x \; \phi(\vec{x}) e^{i\vec{q}\cdot\vec{x}}$$

$$R(E_1, E_2) = (2\pi)^3 2c\hbar \frac{V}{v_a^2} \sum_G \frac{d\Phi}{dE_a}(E_a) \frac{1}{|\vec{G}|^2} \frac{g_{a\gamma\gamma}^2}{16\pi^2} |\sum_j F_{a,j}^0(\vec{G}) S_j(\vec{G})|^2 \sin^2(2\theta) \; \mathcal{W}$$

k : Momentum of axion
q : momentum transferred to the crystal
Φ : electrostatic field potential whether a single nucleus or the entire crystal.

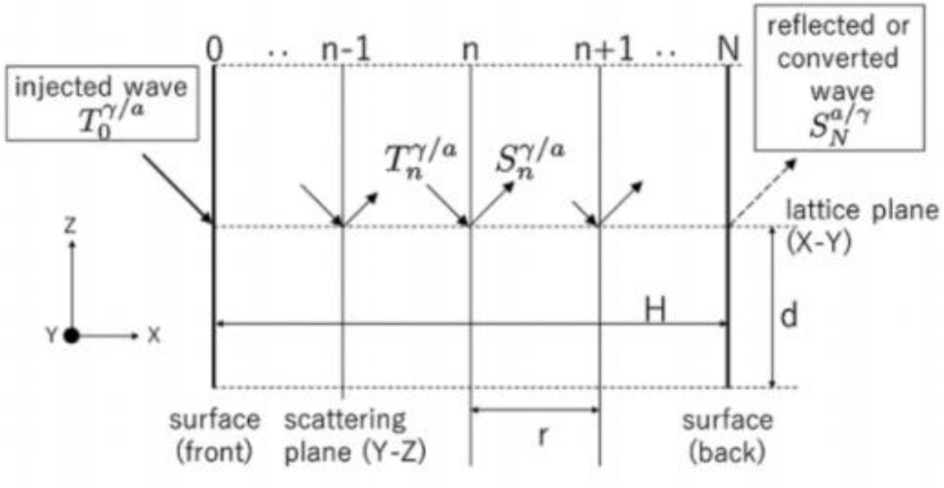
Parameters:
1- V : Crystal volume.
2- v_a: Crystal unit cell.
3- $\frac{d\Phi}{dE_a}$: Axion flux
4- G : Crystal reciprocal lattice vector.
5- $g_{a\gamma\gamma}$: coupling constant.
6- S(G) : Crystal structure function.
7- $F^0(G)$: Interaction form factor.
8- θ: Scattering angle.
9- W: Function of detector resolution.

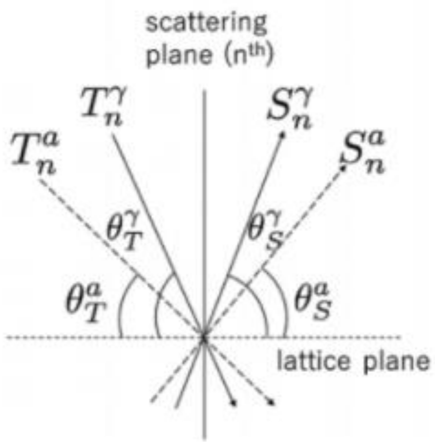
Bragg condition →

$$E_{Bragg} = \hbar c \frac{|G|^2}{2S.G}$$

Axion-photon conversion rate

Dividing the crystal into scattering planes and finding recurrence relations between reflected and transmitted axion/photon waves at each plane.





$$T^\gamma_{n+1} = T^\gamma_n(1 + i\eta_{T0})e^{-i\phi^\gamma_T} + S^\gamma_{n-1}(i\eta_T)e^{-i(\phi^\gamma_S+\phi^\gamma_T)} + S^a_{n-1}(i\zeta'_{ST})e^{-i(\phi^a_S+\phi^\gamma_T)},$$

$$S^\gamma_n = T^\gamma_n(i\eta_S) + S^\gamma_{n-1}(1 + i\eta_{S0})e^{-i\phi^\gamma_S} + T^a_n(i\zeta'_{TS})$$

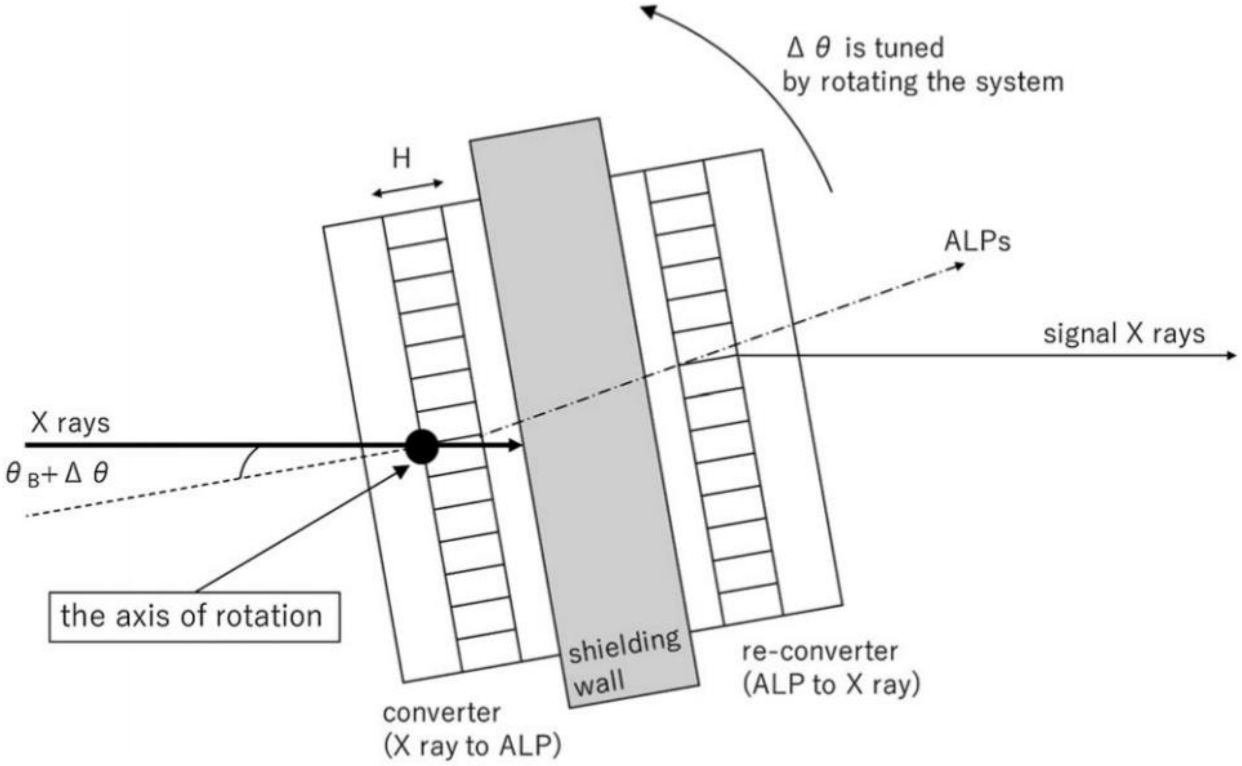
$$T^a_{n+1} = T^a_n e^{-i\phi^a_T} + S^\gamma_{n-1}(i\zeta_{ST})e^{-i(\phi^\gamma_S+\phi^a_T)},$$

$$S^a_n = S^a_{n-1}e^{-i\phi^a_S} + T^\gamma_n(i\zeta_{TS}).$$

arXiv:1709.03299

Axion-photon conversion rate

- This method provides the ratio of $\frac{Final\ wave}{Initial\ wave}$ not the event rate inside the crystal.
- Good for shining through the wall experiments.



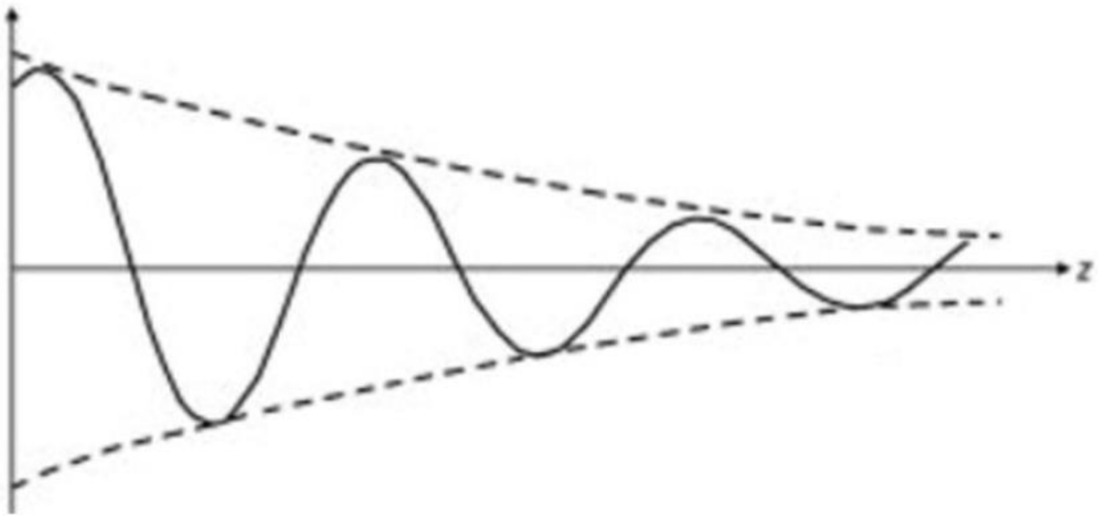
arXiv:1709.03299

Electromagnetic waves in a medium

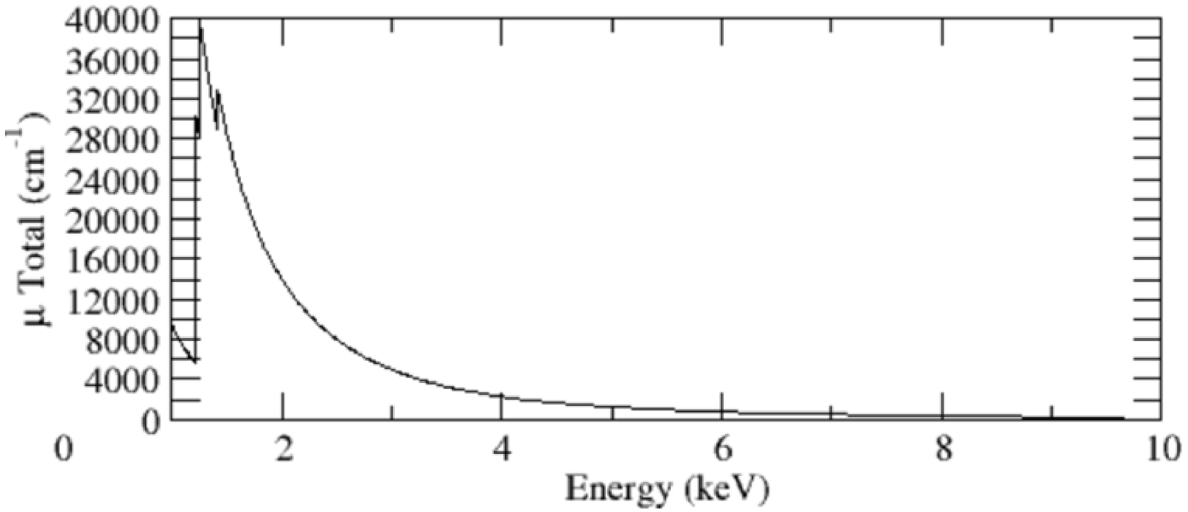
$\boldsymbol{E} = \mathbb{G}(\boldsymbol{k}, \omega) \cdot \boldsymbol{j}$	Electric field (G is the Green function)

$$e^{ikz} = e^{i\left(\frac{\omega}{c}\right)(n'+in'')z} = e^{i\left(\frac{\omega}{c}\right)n'z}e^{-\left(\frac{\omega}{c}\right)n''z}$$

$$I(z) \propto e^{-2\left(\frac{\omega}{c}\right)n''z} = e^{-\mu z}$$



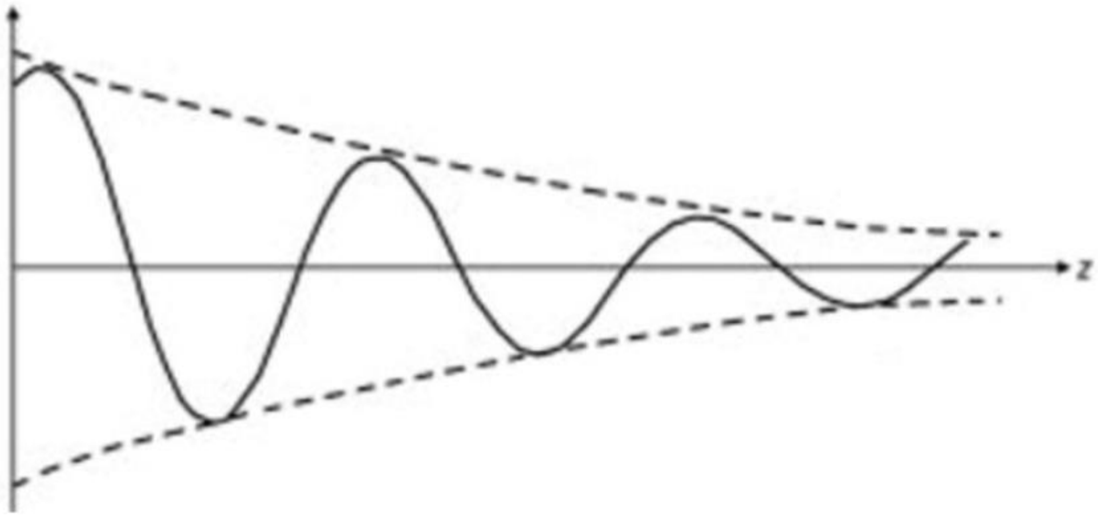
Z = 32, E = 1 - 10 keV

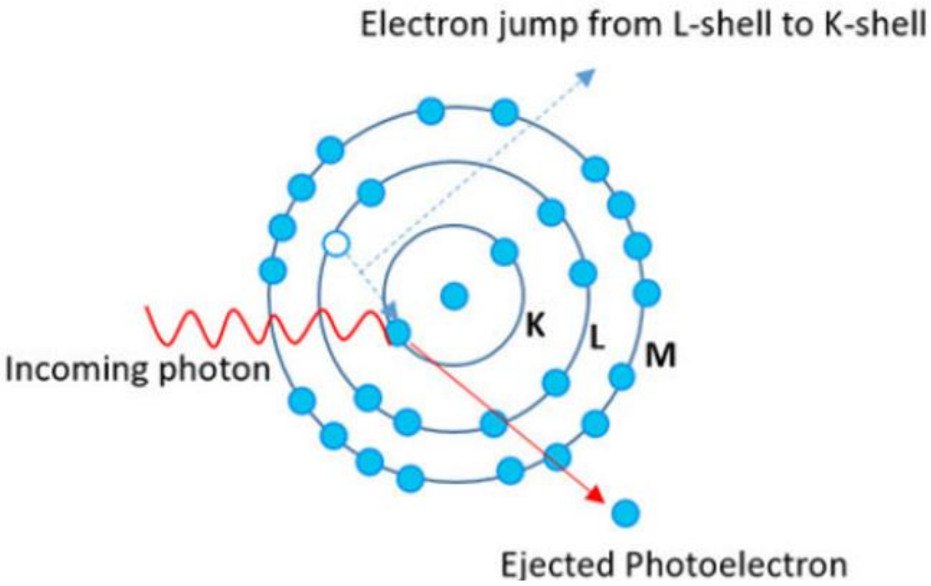


NIST

X-ray absorption

- (1-10 KeV) Photons after conversion -> photoelectric scattering
- Absorption : Axion -> spectrum of photons from a single axion.
- No absorption : Axion -> Single photon (In reality no event but could be imagined to cause a single event.)
- Conventional method doesn’t account for absorption.





Simple example: nucleus

The new method (Including absorption)

Given the Green function:

$$\boldsymbol{j}(\boldsymbol{x},t) = \frac{\eta}{M}\boldsymbol{\mathcal{E}}(\boldsymbol{x}) \times \nabla\phi(\boldsymbol{x},t)$$

$$\boldsymbol{E} = \mathbb{G}(\boldsymbol{k},\omega)\cdot\boldsymbol{j}$$

$$\longrightarrow \quad \frac{d\mathscr{E}}{dt} = -\int \langle \boldsymbol{j}(\boldsymbol{x},t)\cdot\boldsymbol{E}(\boldsymbol{x},t)\rangle\, d\boldsymbol{x}$$

Detectors event rate:

$$R(E_1,E_2) = \int_{E_1}^{E_2} dE_{ee}\int_0^\infty dE_\gamma \frac{dR}{dE_\gamma}(E_\gamma)\frac{1}{\Delta\sqrt{2\pi}}e^{-\frac{(E_{ee}-E_\gamma)^2}{2\Delta^2}}$$

We need the differential production rate of photons at E_γ ($\frac{dR}{dE_\gamma}$)

$$\frac{d^2\varepsilon}{dE_\gamma dt} = E_\gamma\frac{d^2N}{dE_\gamma dt} = E_\gamma\frac{dR(E_\gamma)}{dE_\gamma} \quad\longrightarrow\quad \frac{d\varepsilon}{dt} = \int_0^\infty E_\gamma\frac{dR(E_\gamma)}{dE_\gamma}dE_\gamma$$

The new method (Including absorption)

Differential production rate of photons E_γ ($\frac{dR}{dE_\gamma}$) :

$$\frac{dR}{dE_\gamma} = \frac{\Gamma}{\hbar c}\sum_{\mathbf{G}}\Theta((\hat{\mathbf{s}}\cdot\mathbf{G})^2 - G^2 + k^2)\left[\frac{\mathcal{M}(E_+)\Theta(E_+)}{\hbar ck + E_+} + \frac{\mathcal{M}(E_-)\Theta(E_-)}{\hbar ck + E_-}\right]\frac{k^2}{\sqrt{(\hat{\mathbf{s}}\cdot\mathbf{G})^2 - G^2 + k^2}}$$

$$E_\pm(k) = \hbar c(\hat{\mathbf{s}}\cdot\mathbf{G}) \pm \hbar c\sqrt{(\hat{\mathbf{s}}\cdot\mathbf{G})^2 - G^2 + k^2}$$

$$\mathcal{M}(E, k, \mathbf{G}, \hat{\mathbf{s}}) = \mathcal{F}(E)\ f(\hat{\mathbf{k}}_{\mathbf{G},\hat{\mathbf{s}}}(E), \mathbf{G}, \hat{\mathbf{s}})\ L(k, E/\hbar)$$

- The no-absorption limit is when "skin depth" 1/μ -> ∞. In this limit $L(k,\omega) \to \delta(k - \frac{\omega}{c})$ so, we get a mono-energetic photon per axion as we expected.
- Note that in the no-absorption limit we could reproduce the conventional event rate.

$$R(E_1, E_2) = (2\pi)^3 2c\hbar\frac{V}{v_a^2}\sum_G \frac{d\Phi}{dE_a}(E_a)\frac{1}{|\vec{G}|^2}\frac{g_{a\gamma\gamma}^2}{16\pi^2}|\sum_j F_{a,j}^0(\vec{G})S_j(\vec{G})|^2\sin^2(2\theta)\ \mathcal{W}$$

Parameters:

1. Γ : Constant.
2. F(E) : Flux of axions depending on the axion energy.
3. K : Photon momentum.
4. f : A function of the crystal structure factor, and the interaction form factor.
5. L : A function capturing the absorption length.

$$L(k,\omega) = \frac{1}{\pi}\frac{\frac{\mu(\omega)}{2}}{(k - \frac{\omega}{c})^2 + \frac{\mu^2(\omega)}{4}}$$

$$\boldsymbol{\kappa}_{\boldsymbol{G},\hat{\mathbf{s}}}(E) \equiv \boldsymbol{G} + \frac{E}{\hbar c}\hat{\mathbf{s}}$$

Conservation of momentum

Numeric calculation difficulties

To find the event rate including absorption we need to get numeric integrals:

$$R(E_1, E_2) = \int_{E_1}^{E_2} dE_{ee} \int_0^{\infty} dE_\gamma \frac{dR}{dE_\gamma}(E_\gamma) \frac{1}{\Delta\sqrt{2\pi}} e^{-\frac{(E_{ee}-E_\gamma)^2}{2\Delta^2}}$$

$$\boxed{\frac{dR}{dE_\gamma} = \frac{\Gamma}{\hbar c} \sum_{\mathbf{G}} \Theta((\hat{\mathbf{s}}\cdot\mathbf{G})^2 - G^2 + k^2) \left[\frac{\mathcal{M}(E_+)\Theta(E_+)}{\hbar ck + E_+} + \frac{\mathcal{M}(E_-)\Theta(E_-)}{\hbar ck + E_-}\right] \frac{k^2}{\sqrt{(\hat{\mathbf{s}}\cdot\mathbf{G})^2 - G^2 + k^2}}}$$

Integrand has sharp peaks around Bragg energies. To avoid numerical integration instabilities, we modified the integration limits.

$$R(E_1, E_2) = \int_{E_1}^{E_2} dE_{ee} \sum_{E_{Bragg}} \int_{E_{Bragg}-\delta}^{E_{Bragg}+\delta} dE_\gamma \frac{dR(E_\gamma)}{dE_\gamma} \frac{1}{\Delta\sqrt{2\pi}} e^{-\frac{(E_{ee}-E_\gamma)^2}{2\Delta^2}}$$

Further we tested different integration methods to find the best option.

Integration methods:
1- Simpson's rule. (Big errors)
2- Gaussian quadrature. (Missing events)
3- GSL-QAGS adaptive integration with singularities. (Big errors)
4- GSL-QAG adaptive integration. (Big errors)
5- GSL-CQUAD doubly-adaptive integration. (Best between adoptive methods)