# Chart of Elementary Particles

## Standard Model of Fundamental Particles and Interactions

The Standard Model summarizes the current knowledge in Particle Physics. It is the quantum theory that includes the theory of strong interactions (quantum chromodynamics or QCD) and the unified theory of weak and electromagnetic interactions (electroweak). Gravity is not included in this chart because it is one of the fundamental interactions even though not part of the "Standard Model."

### FERMIONS

<table>
<thead>
<tr>
<th>Flavor</th>
<th>Mass GeV/c²</th>
<th>Electric Charge</th>
</tr>
</thead>
<tbody>
<tr>
<td>ν_e</td>
<td>&lt;1.6×10⁻⁵</td>
<td>0</td>
</tr>
<tr>
<td>ν_μ</td>
<td>&lt;0.002</td>
<td>0</td>
</tr>
<tr>
<td>ν_τ</td>
<td>0.106</td>
<td>0</td>
</tr>
<tr>
<td>e⁻</td>
<td>9.10938</td>
<td>-1</td>
</tr>
<tr>
<td>μ⁻</td>
<td>1.00137</td>
<td>-1</td>
</tr>
<tr>
<td>τ⁻</td>
<td>1.7771</td>
<td>-1</td>
</tr>
</tbody>
</table>

Spins are the intrinsic angular momenta of particles. Spin is given in units of ħ, which is the reduced unit of angular momentum, where ħ = 3h/2π and h = 1.05454 × 10⁻³⁴ J·s. The mass of the proton is 938 MeV/c² and the electric charge of the proton is 1.602 × 10⁻¹⁹ coulombs.

### BOSONS

<table>
<thead>
<tr>
<th>Name</th>
<th>Mass GeV/c²</th>
<th>Electric Charge</th>
</tr>
</thead>
<tbody>
<tr>
<td>γ</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>W⁻</td>
<td>80.4</td>
<td>-1</td>
</tr>
<tr>
<td>W⁺</td>
<td>80.4</td>
<td>+1</td>
</tr>
<tr>
<td>Z⁰</td>
<td>91.167</td>
<td>0</td>
</tr>
</tbody>
</table>

**Color Charge**

Each quark carries one of three types of "strong charge," which are called color charges. There are eight possible pairs of color charges for quarks, and consequently, all quark color-charged particles are exchanged in strong interactions. Quarks and antiquarks can form mesons, which are color singlets, and baryons, which are color octets.

### Residual Strong Interaction

The strong binding of color neutral protons and neutrons to form nuclei is due to residual strong interactions between their color-charged constituents. It is similar to the residual electromagnetic interaction that binds electrically neutral atoms to form molecules. It can also be viewed as the exchange of mesons between the hadrons.

### Properties of the Interactions

- **Gravitational Interaction**
- **Weak Interaction**
  - All Particles and Leptons: W⁻, W⁺, Z⁰
  - Electromagnetically Charged Particles: Quarks, Gluons
  - Weak Neutral Bosons: Hadrinos

### Baryons and Antibaryons

- **Symbol**
- **Name**
- **Quark Content**
- **Electric Charge**
- **Mass (in GeV)**
- **Type**

### Matter and Antimatter

For every particle, there is a corresponding antiparticle, denoted by a bar over the particle symbol (e.g., p → p̅). Particles and antiparticles have identical mass and spin but opposite charge. Some electrically neutral bosons (e.g., Z⁻, W⁺, and Z⁰) are not their own antiparticles.

### Figures

- **Figures**
  - A neutron decays into a proton, an electron, and an antineutrino via a virtual (mediating) W⁻ boson. This is related to the process of beta decay.
  - An electron and positron annihilate at high energies, cooling down and producing a virtual W⁻ and W⁺ boson via a virtual Z⁰ boson or a virtual photon.
  - Two protons colliding at high energy can produce massive final states like the Higgs boson, which is believed to give mass to other particles.

The **Particle Adventure** is an interactive website that explores the Standard Model of Particles and Field Theory. It offers a wealth of information on the fundamental forces, particles, and their interactions, using interactive tools and animations.
**Chart of Elementary Particles**

### Fermions

<table>
<thead>
<tr>
<th>Flavor</th>
<th>Mass GeV/c²</th>
<th>Electric charge</th>
<th>Spin</th>
</tr>
</thead>
<tbody>
<tr>
<td>e⁻ electron</td>
<td>0.000511</td>
<td>-1</td>
<td>1/2</td>
</tr>
<tr>
<td>νₑ electron neutrino</td>
<td>&lt;1×10⁻⁸</td>
<td>0</td>
<td>1/2</td>
</tr>
<tr>
<td>μ⁻ muon</td>
<td>0.106</td>
<td>0</td>
<td>1/2</td>
</tr>
<tr>
<td>νₘ muon neutrino</td>
<td>&lt;0.0002</td>
<td>0</td>
<td>1/2</td>
</tr>
<tr>
<td>νₜ tau neutrino</td>
<td>&lt;0.02</td>
<td>0</td>
<td>1/2</td>
</tr>
<tr>
<td>τ⁻ tau</td>
<td>1.777</td>
<td>0</td>
<td>1/2</td>
</tr>
</tbody>
</table>

### Quarks

<table>
<thead>
<tr>
<th>Flavor</th>
<th>Mass GeV/c²</th>
<th>Electric charge</th>
<th>Spin</th>
</tr>
</thead>
<tbody>
<tr>
<td>u up</td>
<td>0.003</td>
<td>2/3</td>
<td>1/2</td>
</tr>
<tr>
<td>d down</td>
<td>0.006</td>
<td>-1/3</td>
<td>1/2</td>
</tr>
<tr>
<td>c charm</td>
<td>1.3</td>
<td>2/3</td>
<td>1/2</td>
</tr>
<tr>
<td>s strange</td>
<td>0.1</td>
<td>-1/3</td>
<td>1/2</td>
</tr>
<tr>
<td>t top</td>
<td>4.3</td>
<td>2/3</td>
<td>1/2</td>
</tr>
</tbody>
</table>

### Mesons

<table>
<thead>
<tr>
<th>Quark pair</th>
<th>Mass GeV/c²</th>
<th>Electric charge</th>
<th>Spin</th>
</tr>
</thead>
<tbody>
<tr>
<td>u⁺d⁻</td>
<td>0.146</td>
<td>0</td>
<td>1/2</td>
</tr>
<tr>
<td>s⁺s⁻</td>
<td>0.544</td>
<td>0</td>
<td>1/2</td>
</tr>
<tr>
<td>u⁺d⁺</td>
<td>0.373</td>
<td>1</td>
<td>0</td>
</tr>
<tr>
<td>d⁺s⁻</td>
<td>1.229</td>
<td>0</td>
<td>1/2</td>
</tr>
<tr>
<td>u⁺c⁻</td>
<td>2.580</td>
<td>0</td>
<td>1/2</td>
</tr>
</tbody>
</table>

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**Figure**

These diagrams are an artist's conception of physical phenomena. They are not exact and have no meaningful scale. Green shaded areas represent the cloud of gluons in the gluon field, and red lines the quark paths.

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**Notes**

- A neutron decays to a proton, an electron, and an antineutrino via a virtual pair of bosons. This is neutron decay.
- An electron and positron pair can combine at high energy to produce a Higgs and a Z boson, or a virtual boson or a virtual photon.
- Two protons colliding at high energy can produce a meson pair in a very high mass state that decays very quickly into two quarks that are one but can exist only due to the structure of matter.

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**Beistle Ind., Inc.**

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Better Chart!

**FUNDAMENTAL PARTICLES AND INTERACTIONS**

The Standard Model is a quantum theory that summarizes our current knowledge of the physics of fundamental particles and fundamental interactions (interactions are mediated by force and by decay rates of unstable particles).

### BOSONS
- **Quarks and Gluons**
  - Quarks carry "strong" color charge and can form strong interactions.
  - Each quark carries three types of color charge.
  - These charges have nothing to do with the colors of visible light.
  - Just as electrically-charged particles interact by exchanging photons, in strong interactions, color-charged particles interact by exchanging gluons.

### FERMIONS
- **Leptons**
  - Spin = 1/2, massless, and electrically neutral.
- **Quarks**
  - Spin = 1/2, massless, and electrically neutral.

### Structure within the Atom
- Quark: Size = 10^{-15} m
- Electron: Size = 10^{-18} m
- Proton: Size = 10^{-15} m

### Properties of the Interactions
- **Gravitational Interaction**
- **Weak Interaction** (Electroweak) **Electromagnetic Interaction** **Strong Interaction**

### Mysterious Particles
- **Unseen Particles**
  - Driven by new puzzles in our understanding of the physical world, particle physicists are following paths to new workers and startling discoveries. Experiments may even find extra dimensions of space, mini-black holes, and/or evidence of string theory.

### Unsolved Mysteries
- **Origin of Mass?**
- **Dark Matter?**
- **Why No Antimatter?**
- **Universe Accelerating?**

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**Particle Adventure**
- Visit the award-winning website: [The Particle Adventure at ParticleAdventure.org](http://ParticleAdventure.org)

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**CPEWeb.org**
- A project of the University of California, Lawrence Berkeley National Laboratory, and the U.S. Department of Energy.
- One of a series of websites funded by the National Science Foundation and others.

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**See the neutrino paragraph below.**

Spin is the intrinsic angular momentum of particles. Spin is given in units of ħ, which is the quantum unit of angular momentum where ħ = h/2π = 6.63x10^{-34} GeV·s = 1.05x10^{-34} J·s.

**Electric charges** are given in units of the proton's charge. In SI units the electric charge of the proton is 1.602x10^{-19} C.

**The energy** of a particle is given by the kinetic energy (E) of the particle. This energy is given by E = mc², where E is the energy (J), c is the speed of light (3x10^8 m/s), and m is the mass (kg) of the particle. The energy of the proton is 938.29 GeV (2.19x10^{-12} kg).

**Neutral** Neutrinos are produced in the sun, supernovae, reactors, accelerators, and many other processes. They are produced in pairs, and the electron neutrino can be described as one of three neutrino flavors: μν, τν, or νe. Each flavor is related to a type of charged particle (mesons, μ, τ, and e). The neutrinos are produced in pairs, and each flavor is related to a type of charged particle (mesons, μ, τ, and e).

**Particle and Antiparticle**
- For every particle type there is a corresponding antiparticle type. Parity is conserved if it is one of the six known antiparticles: (μ, τ, and e). Parity is also conserved if it is one of the six known antiparticles: (μ, τ, and e).
- The antiparticle of a particle is given by changing the sign of the mass and spin. The antiparticle of a charged particle is given by changing the sign of the electric charge.

**These diagrams are an artist's conception. Blue-green shaded areas represent the cloud of quarks.**

- A free neutron (n) always decays to a proton (p) and an electron (e⁻) and a neutrino (v). This is neutron (n) decay.
- An electron and positron (e⁺) collide at high energy and annihilate to produce 2γ (gamma) photons. This is electron (e⁻) positron (e⁺) annihilation.
**FERMIONS**

**Leptons**  
<table>
<thead>
<tr>
<th>Flavor</th>
<th>Mass GeV/c²</th>
<th>Electric charge</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\nu_L$ (lightest neutrino)</td>
<td>$(0-0.13) \times 10^{-9}$</td>
<td>0</td>
</tr>
<tr>
<td>$e$ (electron)</td>
<td>$0.000511$</td>
<td>$-1$</td>
</tr>
<tr>
<td>$\nu_M$ (middle neutrino)</td>
<td>$(0.009-0.13) \times 10^{-9}$</td>
<td>0</td>
</tr>
<tr>
<td>$\mu$ (muon)</td>
<td>$0.106$</td>
<td>$-1$</td>
</tr>
<tr>
<td>$\nu_H$ (heaviest neutrino)</td>
<td>$(0.04-0.14) \times 10^{-9}$</td>
<td>0</td>
</tr>
<tr>
<td>$\tau$ (tau)</td>
<td>$1.777$</td>
<td>$-1$</td>
</tr>
</tbody>
</table>

**Quarks**  
<table>
<thead>
<tr>
<th>Flavor</th>
<th>Approx. Mass GeV/c²</th>
<th>Electric charge</th>
</tr>
</thead>
<tbody>
<tr>
<td>$u$ (up)</td>
<td>$0.002$</td>
<td>$2/3$</td>
</tr>
<tr>
<td>$d$ (down)</td>
<td>$0.005$</td>
<td>$-1/3$</td>
</tr>
<tr>
<td>$c$ (charm)</td>
<td>$1.3$</td>
<td>$2/3$</td>
</tr>
<tr>
<td>$s$ (strange)</td>
<td>$0.1$</td>
<td>$-1/3$</td>
</tr>
<tr>
<td>$t$ (top)</td>
<td>$173$</td>
<td>$2/3$</td>
</tr>
<tr>
<td>$b$ (bottom)</td>
<td>$4.2$</td>
<td>$-1/3$</td>
</tr>
</tbody>
</table>

**Matter Constituents**  
spin = 1/2, 3/2, 5/2, ...
As of today: Oscillation of 3 massive active neutrinos is clearly the dominant effect:

If neutrinos have mass: \[ |\nu_l\rangle = \sum U_{li} |\nu_i\rangle \]

For 3 Active neutrinos.

\[
U_{li} = \begin{pmatrix}
U_{e1} & U_{e2} & U_{e3} \\
U_{\mu1} & U_{\mu2} & U_{\mu3} \\
U_{\tau1} & U_{\tau2} & U_{\tau3}
\end{pmatrix}
\]

\[
= \begin{pmatrix}
1 & 0 & 0 \\
0 & c_{23} & s_{23} \\
0 & -s_{23} & c_{23}
\end{pmatrix} \cdot \begin{pmatrix}
1 & 0 & 0 \\
0 & 1 & 0 \\
0 & 0 & e^{-i\delta}
\end{pmatrix} \cdot \begin{pmatrix}
c_{13} & 0 & s_{13} \\
0 & 1 & 0 \\
-s_{13} & 0 & c_{13}
\end{pmatrix} \cdot \begin{pmatrix}
c_{12} & s_{12} & 0 \\
-s_{12} & c_{12} & 0 \\
0 & 0 & 1
\end{pmatrix} \cdot \begin{pmatrix}
1 & 0 & 0 \\
0 & e^{-i\alpha_2/2} & 0 \\
0 & 0 & e^{-i\alpha_1/2+i\delta}
\end{pmatrix}
\]

Pontecorvo-Maki-Nakagawa-Sakata matrix

(Double \(\beta\) decay only)

Atmospheric, Accel. CP Violating Phase Reactor, Accel. Solar, Reactor Majorana CP Phases

where \(c_{ij} = \cos \theta_{ij}\), and \(s_{ij} = \sin \theta_{ij}\)

For two neutrino oscillation in a vacuum: (a valid approximation in many cases)

\[ P(\nu_\mu \rightarrow \nu_e) = \sin^2 2\theta \sin^2 \left(1.27 \frac{\Delta m^2 L}{E}\right) \]

CP Violating Phase or Majorana Phases: Antimatter/matter asymmetry in Early Universe?
As of today: Oscillation of 3 massive active neutrinos is clearly the dominant effect:

If neutrinos have mass: $|\nu_l\rangle = \sum U^*_{li} |\nu_i\rangle$

For 3 Active neutrinos.

$U_{li} = \begin{pmatrix} U_{e1} & U_{e2} & U_{e3} \\ U_{\mu1} & U_{\mu2} & U_{\mu3} \\ U_{\tau1} & U_{\tau2} & U_{\tau3} \end{pmatrix}$

= \begin{pmatrix} 1 & 0 & 0 \\ 0 & c_{23} & s_{23} \\ 0 & -s_{23} & c_{23} \end{pmatrix} \cdot \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & e^{-i\delta} \end{pmatrix} \cdot \begin{pmatrix} c_{13} & 0 & s_{13} \\ 0 & 1 & 0 \\ -s_{13} & 0 & c_{13} \end{pmatrix} \cdot \begin{pmatrix} c_{12} & s_{12} & 0 \\ -s_{12} & c_{12} & 0 \\ 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} 1 & 0 & 0 \\ 0 & e^{-i\alpha_l/2} & 0 \\ 0 & 0 & e^{-i\alpha_r/2+i\delta} \end{pmatrix}$

\text{(Double } \beta \text{ decay only)}

Atmospheric, Accel. CP Violating Phase Reactor, Accel. Solar, Reactor Majorana CP Phases

where $c_{ij} = \cos \theta_{ij}$, and $s_{ij} = \sin \theta_{ij}$

For two neutrino oscillation in a vacuum: (a valid approximation in many cases)

$$P(\nu_\mu \rightarrow \nu_e) = \sin^2 2\theta \sin^2 \left(1.27 \frac{\Delta m^2 L}{E} \right)$$

CP Violating Phase or Majorana Phases: Antimatter/matter asymmetry in Early Universe?
Neutrinos Oscillate
to thus they have mass

- flux of atmospheric muon neutrinos produced by cosmic rays is not up-down symmetric
- solar neutrinos produced as electron neutrinos in the Sun are detected by SNO as other flavours ($\nu_\mu$, $\nu_\tau$)
To be complete…

- we’ve also seen the disappearance of reactor antineutrinos due to oscillations at long baselines (~180 km) and short baselines (~1 km)
- we’ve also seen the disappearance of accelerator-produced beams of $\nu_\mu$ and also their appearance downstream as $\nu_e$ and $\nu_\tau$

_We know neutrinos oscillate – they can change flavour as they propagate!_
Neutrino Oscillations

- flavour eigenstates and mass eigenstates mix in the lepton sector, like the quarks do

\[
\nu_f = \sum_i U_{fi} \nu_i
\]

simplified expressions for two-flavour mixing:

\[
\nu_e = \nu_1 \cos \theta + \nu_2 \sin \theta \\
\nu_\mu = -\nu_1 \sin \theta + \nu_2 \cos \theta
\]

\[
P_{e\mu} = \sin^2 2\theta \sin^2 \frac{1.267 \Delta m^2 L}{E}
\]

\(\Delta m^2\) in \([\text{eV}^2]\), \(E\) in \([\text{MeV}]\), \(L\) in \([\text{m}]\)

where \(\Delta m^2 = m_2^2 - m_1^2\)
Characteristic Oscillation Length

\[ P_{e\mu} = \sin^2 2\theta \sin^2 \frac{1.267 \Delta m^2 L}{E} \]

\[ \frac{1.267 \Delta m^2 L_{osc}}{E} = \pi \]

\[ L_{osc} [\text{m}] = \frac{\pi}{1.267} \frac{E [\text{MeV}]}{\Delta m^2 [\text{eV}^2]} \]

calculate a few of these for yourself: KamLAND reactor neutrinos, T2K long baseline GeV neutrinos, Daya Bay reactor neutrinos
Typical 2-$\nu$ Oscillation Result

**CHOOZ**

Reactor $\bar{\nu}_e$

Disappearance
Schrödinger’s Cat

• Neutrino oscillations is like Schrödinger’s Cat™

Hmm… wait, I get it!

The neutrino wavefunction is simultaneously $\nu_1$ and $\nu_2$ as it propagates!
Young’s Two-Slit Experiment

- Neutrino oscillations is like the two-slit experiment!

If I measure which neutrino mass eigenstate was produced, I will get a “single-slit pattern”.

If I don’t measure which mass eigenstate was emitted in the charged-current reaction, both are involved and I will get a “two-slit” interference pattern.

That’s neutrino oscillations!
Quark Mixing

- CKM – Cabibbo-Kobayashi-Maskawa matrix describes quark flavour mixing
  - we think of this slightly differently than we usually do for leptons

from Wikipedia

into up quarks ($|V_{ud}|^2$ and $|V_{us}|^2$ respectively). In particle physics parlance, the object that couples to the up quark via charged-current weak interaction is a superposition of down-type quarks, here denoted by $d'$.\[^4\] Mathematically this is:

$$d' = V_{ud}d + V_{us}s,$$

or using the Cabbibo angle:

$$d' = \cos \theta_c d + \sin \theta_c s.$$
Charged-Current Interactions with Quarks

- top quarks often decay to bottom quarks, sometimes to strange quarks, very occasionally to down quarks
  - nobody has a problem with this!

- bottom quarks decay (undergo charged-current interactions that transform them) into charm quarks or sometimes up quarks
  - nobody has a problem with this!

Translate into Neutrino Language

- muons undergo charged-current interactions sometimes into $\nu_1$, sometimes to $\nu_2$, and sometimes to $\nu_3$
- if we have a $\nu_2$ state propagating, it can undergo a charged-current interaction that could transform it into an electron, muon (or a tau, if energetic enough)

Perfectly analogous!
Why Oscillations?

• If we don’t know whether it is a $\nu_2$ state or a $\nu_1$ state that is propagating, we have to consider that it is both, mixed as appropriate for the way the states were produced, coherent if produced that way, and propagating with different phases for the mass eigenstates, interfering with each other.

• The combination $\nu_2$ state and $\nu_1$ state can undergo a charged-current interaction transforming it into an electron, muon, or tau…depending on the coherent superposition of the possibilities for each of the $\nu_2$ state and $\nu_1$ state (which depends on their phases at that instant).

• *It takes some words to say correctly…but, if you understand the above, you’ve understood neutrino oscillations completely!*
So, the Next Time Somebody Asks You…

- **why** do neutrinos oscillate?
- why don’t electrons and muons “oscillate”?
- why don’t quarks “oscillate”?
  - or do they?
- …you will be able to answer!
Three-Flavour Neutrino Oscillations (in vacuum, plane-wave model)

- I was going to write this on the white board…
- then, thought I’d LaTeX it up for PowerPoint…
- then, decided, let’s just cut and paste from Giunti and cite him

\[
|\nu_k(x,t)\rangle = e^{-iE_k t + ip_k x} |\nu_k\rangle \quad \Rightarrow \quad |\nu_\alpha(x,t)\rangle = \sum_k U^*_{\alpha k} e^{-iE_k t + ip_k x} |\nu_k\rangle
\]

\[
|\nu_\alpha(x,t)\rangle = \sum_{\beta=e,\mu,\tau} \left( \sum_k U^*_{\alpha k} e^{-iE_k t + ip_k x} U_{\beta k} \right) |\nu_\beta\rangle
\]

\[A_{\nu_\alpha \rightarrow \nu_\beta}(x,t)\]

Transition Probability

\[
P_{\nu_\alpha \rightarrow \nu_\beta}(x,t) = |\langle \nu_\beta | \nu_\alpha(x,t) \rangle|^2 = |A_{\nu_\alpha \rightarrow \nu_\beta}(x,t)|^2 = \left| \sum_k U^*_{\alpha k} e^{-iE_k t + ip_k x} U_{\beta k} \right|^2
\]
Three-Flavour Oscillations, cont’d

ultrarelativistic neutrinos $\implies t \simeq x = L$ source-detector distance

$$E_k t - p_k x \simeq (E_k - p_k) L = \frac{E_k^2 - p_k^2}{E_k + p_k} L \simeq \frac{m_k^2}{E_k + p_k} L \propto \frac{m_k^2}{2E} L$$

$$P_{\nu_\alpha \to \nu_\beta}(L, E) = \left| \sum_k U_{\alpha k}^* e^{-i m_k^2 L/2E} U_{\beta k} \right|^2$$

$$= \sum_k |U_{\alpha k}|^2 |U_{\beta k}|^2$$

$$+ 2 \Re \sum_{k > j} U_{\alpha k}^* U_{\beta k} U_{\alpha j} U_{\beta j}^* \exp \left( -i \frac{\Delta m_{k,j}^2 L}{2E} \right) \Leftrightarrow \text{constant term}$$

$$\Delta m_{k,j}^2 \equiv m_k^2 - m_j^2$$

$$\Leftrightarrow \text{oscillating term}$$

$\uparrow \downarrow \text{coherence}$

Giunti
Neutrinos and Antineutrinos

Antineutrinos are described by CP-conjugated fields:

$$\nu^{CP} = \gamma^0 \mathcal{C} \bar{\nu}^T = -\mathcal{C} \nu^*$$

- C $\iff$ Particle $\iff$ Antiparticle
- P $\iff$ Left-Handed $\iff$ Right-Handed

Fields:

$$\nu_{\alpha L} = \sum_k U_{\alpha k} \nu_{kL} \quad \overset{CP}{\longrightarrow} \quad \nu_{\alpha L}^{CP} = \sum_k U_{\alpha k}^* \nu_{kL}^{CP}$$

States:

$$|\nu_{\alpha}\rangle = \sum_k U_{\alpha k}^* |\nu_k\rangle \quad \overset{CP}{\longrightarrow} \quad |\bar{\nu}_{\alpha}\rangle = \sum_k U_{\alpha k} |\bar{\nu}_k\rangle$$

**NEUTRINOS** $U \iff U^*$ **ANTINEUTRINOS**

$$P_{\nu_{\alpha} \rightarrow \nu_{\beta}} (L, E) = \sum_k |U_{\alpha k}|^2 |U_{\beta k}|^2 + 2\text{Re} \sum_{k>j} U_{\alpha k}^* U_{\beta k} U_{\alpha j} U_{\beta j}^* \exp \left( -i \frac{\Delta m_{kj}^2 L}{2E} \right)$$

$$P_{\bar{\nu}_{\alpha} \rightarrow \bar{\nu}_{\beta}} (L, E) = \sum_k |U_{\alpha k}|^2 |U_{\beta k}|^2 + 2\text{Re} \sum_{k>j} U_{\alpha k} U_{\beta k}^* U_{\alpha j}^* U_{\beta j} \exp \left( -i \frac{\Delta m_{kj}^2 L}{2E} \right)$$

Giunti
Neutrinos and Antineutrinos

Antineutrinos are described by CP-conjugated fields:

\[ \nu^T = -C \nu^* \]

C → Particle ↔ Antiparticle
P → Left-Handed ↔ Right-Handed

Fields: \( \nu_{\alpha L} = \sum_k U_{\alpha k} \nu_k \)

States: \( |\nu_{\alpha L}\rangle = \sum_k U_{\alpha k} |\nu_k\rangle \)

Neutrinos

\[ P_{\nu_{\alpha} \rightarrow \nu_{\beta} (L, E)} \]

\[ P_{\bar{\nu}_{\alpha} \rightarrow \bar{\nu}_{\beta} (L, E)} = |U_{\alpha k}|^2 + 2\text{Re} \sum_{k > j} U_{\alpha k}^* U_{\beta k} U_{\alpha j} U_{\beta j} \exp \left( -i \frac{\Delta m^2_{kj} L}{2E} \right) \]

Giunti
PMNS Neutrino Mixing Matrix

$$v_f = \sum_i U_{fi} v_i$$

Pontecorvo, Maki, Nakagawa, Sakata

\[ N = 3 \implies 3 \text{ Mixing Angles} \quad 1 \text{ Dirac Phase} \quad 2 \text{ Majorana Phases} \]

standard parameterization (convenient)

\[ (c_{ij} \equiv \cos \theta_{ij}, \quad s_{ij} \equiv \sin \theta_{ij}) \]

\[
U = R_{23} W_{13} R_{12} D(\lambda) \\
= \begin{pmatrix}
1 & 0 & 0 \\
0 & c_{23} & s_{23} \\
0 & -s_{23} & c_{23}
\end{pmatrix}
\begin{pmatrix}
c_{13} & 0 & s_{13}e^{-i\delta_{13}} \\
0 & 1 & 0 \\
-s_{13}e^{i\delta_{13}} & 0 & c_{13}
\end{pmatrix}
\begin{pmatrix}
c_{12} & s_{12} & 0 \\
-s_{12} & c_{12} & 0 \\
0 & 0 & 1
\end{pmatrix}
\begin{pmatrix}
1 & 0 & 0 \\
0 & e^{i\lambda_{21}} & 0 \\
0 & 0 & e^{i\lambda_{31}}
\end{pmatrix}
\]

\[
= \begin{pmatrix}
c_{12}c_{13} \\
-s_{12}c_{13} \\
s_{12}c_{13}
\end{pmatrix}
\begin{pmatrix}
s_{12}c_{13} \\
-c_{12}c_{13} \\
s_{12}c_{13}
\end{pmatrix}
\begin{pmatrix}
s_{13}e^{-i\delta_{13}} \\
-c_{12}s_{23}s_{13}e^{i\delta_{13}} \\
c_{12}s_{23}s_{13}e^{i\delta_{13}}
\end{pmatrix}
\begin{pmatrix}
1 & 0 & 0 \\
0 & e^{i\lambda_{21}} & 0 \\
0 & 0 & e^{i\lambda_{31}}
\end{pmatrix}
\]

atmospheric \quad reactor/T2K \quad solar plus KamLAND

Giunti
Being Pedantic – How Many Phases?

3×3 unitary matrix (complex-valued)
9 unitarity equations
= 9 real parameters or 3 angles and 6 phases

- if neutrinos are Dirac fermions, all but one phase can be rotated away in the definition of the fields
- if neutrinos are Majorana fermions, only 3 phases can be absorbed into the definition of the fields

PMNS neutrino mixing matrix, $U$, therefore has either:

- 3 Majorana phases
- or 1 Dirac phase
Being Pedantic – Octant Degeneracy

- what are the possible values of the PMNS matrix elements, $U_{\alpha k}$?
- construction of the full $3 \times 3$ matrix with complex phase is non-trivial…the octant can (does) matter
- oscillation experiments explore $\sin^2 2\theta$, resulting in an octant degeneracy in determining $U_{\alpha k}$

$$P_{e\mu} = \sin^2 2\theta \sin^2 \frac{1.27 \Delta m^2 L}{E}$$
Step Back to $2 \times 2$

- the $2 \times 2$ unitary matrix is trivial

- if $\theta$ is negative (between $\pi$ and $2\pi$), $\cos \theta$ stays the same and $\sin \theta \rightarrow -\sin(-\theta)$, so we can map it to the positive angle and the matrix is just the transpose (no effect on oscillations)

- if $\theta > \pi/2$, $\cos \theta \rightarrow -\cos(-\theta)$, $\sin \theta$ stays the same, so we can map it back to the first quadrant and the matrix is just the transpose, multiplied by $-1$ (no effect)
Angles, Octants, Mass Hierarchy

- if $\theta > \pi/4$, $\cos\theta \rightarrow \sin(\pi/2-\theta) \rightarrow \sin\theta'$
  $\sin\theta \rightarrow \cos(\pi/2-\theta) \rightarrow \cos\theta'$ and then we could map it back to the first octant and it would the same as flipping the mass hierarchy with a relative phase of $e^{i\pi}=-1$ between them.

No effect on 2-flavour, vacuum oscillations…

\[
\begin{pmatrix}
\nu_e \\
\nu_\mu
\end{pmatrix}
= \begin{pmatrix}
\sin\theta' & \cos\theta' \\
-\cos\theta' & \sin\theta'
\end{pmatrix}
\begin{pmatrix}
\nu_1 \\
\nu_2
\end{pmatrix}
\]

\[
\begin{pmatrix}
\nu_e \\
\nu_\mu
\end{pmatrix}
= \begin{pmatrix}
\cos\theta & \sin\theta \\
-\sin\theta & \cos\theta
\end{pmatrix}
\begin{pmatrix}
\nu_1 \\
\nu_2
\end{pmatrix}
\]
Conclusions

- for 2-neutrino mixing, the first octant is sufficient for describing vacuum oscillations, without loss of generality
- the second octant is equivalent to flipping the mass hierarchy, which an oscillation experiment *in vacuum* can’t determine in any case
  - i.e. the sign of $\Delta m^2$ doesn’t matter

- once we introduce matter effects, the hierarchy does matter and the second octant isn’t degenerate with the first

- you hear all the time that the 2-neutrino approximation is a good one (it is, for what we use it for!); but, we live in a 3-neutrino (or more?!) world and the full treatment does matter when we look at more subtle details