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Magnetic Connection Model for Launching Relativistic Jets from a Kerr Black Hole

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ABSTRACT
We present an alternative model for launching relativistic jets in active galactic nuclei (AGN) from an accreting Kerr black hole (BH) by converting the accretion disc energy into jet energy, when the rotational energy of the BH is transferred to the inner disc by closed magnetic field lines which connects the BH to the disc (BH-disc magnetic connection). In this way, the available disc energy is increased by the BH rotational energy. We assume that the BH may undergo recurring episodes of its activity with: (i) a first phase when accretion power dominates, and (ii) a second phase when BH spin-down power dominates. In both cases the jet is driven by a low-luminosity, (geometrically) thin accretion disc, as the disc energy is used to launch the jet. We use the general relativistic conservation laws to calculate the mass flow rate into the jets, the launching power of the jets, and the angular momentum transported by the jets. We consider BHs with a spin parameter $a_* \geqslant 0.95$, so that the jets are launched from the region inside of the BH ergosphere. The angular momentum removed from the accretion disc is carried away by the disc particles that ultimately form the jets. As far as the BH is concerned, it can (i) spin up by accreting matter and (ii) spin down due to the magnetic counter-acting torque on the BH. We found that a stationary state of the BH ($a_* = \mathrm{const}$) can be reached if the mass accretion rate is larger than $\dot{m} \sim 0.001$. Here, the mass accretion rate $\dot{M}$ is specified in units of Eddington accretion rate $\dot{M}_{\rm Edd}$ by $\dot{m} = \dot{M}/\dot{M}_{\rm Edd}$. The maximum value of the BH spin parameter depends on $\dot{m}$ being less but close to 0.9982 (Thorne's model). For $\dot{m} < 0.001$, the BH spins down continuously, unless a large amount of matter is provided. In addition, the maximum AGN lifetime can be much longer than $\sim 10^7$ yr when using the BH spin-down power. This result is consistent with the estimation of the maximum AGN lifetime when the AGN output power is provided by the Blandford–Znajek mechanism.

Key words: accretion, accretion discs – black hole physics – magnetic fields – galaxies:jets

1 INTRODUCTION

Relativistic jets are highly collimated plasma outflows present in extragalactic radio sources, which are associated with many AGN. The launching power of the jet can generally be provided by the accretion disc, by the BH rotation, or both. Moreover, as the jet is launched, the BH can evolve towards a stationary state with a spin parameter whose maximum value is less but close to one ($a_* \lesssim 1$, where $-1 \leqslant a_* \leqslant +1$). One can consider the launching power of the jet to be a fraction of the disc power. A number of questions come to mind: Is this fraction generally valid for astrophysical jets from BHs with the same mass and spin? Can the disc manage to launch the jet by itself as the BH accretes at low rates? How does the magnetic field get involved? Can the BH take over and support the disc to launch the jet as the mass accretion rate goes down? How does the BH spin evolve while the jet is launched, and what is the maximum spin parameter in this case? We try to answer these questions using the model proposed in this paper.

A supermassive BH ($M \sim 10^9 M_\odot$) can be fed and spun up by accreting matter with a consistent sense of the angular momentum [the first calculations for a Kerr BH were performed by Bardeen (1970)] or by merging with another BH (e.g., Berti & Volonteri 2008; Gergely & Biermann 2009). The general relativistic effects on the structure of the inner regions of an accretion disc surrounding a Kerr BH were first studied by Novikov & Thorne (1973) and Page & Thorne (1974) using Bardeen et al.'s (1972) orthonormal frames of the locally nonrotating observers. These studies resulted in

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a geometrically thin, optically thick accretion disc model [see also Shakura & Sunyaev (1973) for a quasi-Newtonian approach to the description of the disc accretion onto a Schwarzschild BH]. In the inner regions of the disc, the radiation pressure dominates the gas pressure. The opacity is dominated by electron scattering; i.e., the photons random-walk before leaving the disc as they scatter off of electrons. Donea & Biermann (1996) developed a model of thin accretion disc driven jets, which can explain the shape of UV spectra from an AGN when the disc is sub-Eddington. Models of jet/wind formation from an accretion disc typically invoke specific magnetic field structures. The jet can be launched and collimated, for instance, by centrifugal and magnetic forces (e.g., Blandford & Payne 1982). A possible condition for centrifugal launching of jets from a thin accretion disc is that the coronal particles, found just above the disc, should go into unstable orbits around the BH (Lyutikov 2009). The relativistic jets can also be launched from either (i) a geometrically thick disc with an advection-dominated accretion flow (e.g., Narayan & Yi 1994; Armitage & Natarajan 1999; Blandford & Begelman 1999; Meier 2001) or (ii) a layer located between the accretion disc and the BH corona, which consists of a highly diffusive, super-Keplerian rotating and thermally dominated by virial-hot and magnetised ion-plasma (Hujeirat et al. 2002). Falcke & Biermann (1995, 1999) proposed a jet-disc symbiosis model for powering jets; starting from the assumption that radio jets and accretion discs are symbiotic features present in radio quasars, these objects consist of a maximal jet power with a total equipartition (i.e., the magnetic energy flow of the jet is comparable to the kinetic jet power), and the total jet power is a particular fraction of the disc power. This fraction can be found by fitting the jet parameters to the observational data.

The energy and angular momentum of a BH can be electromagnetically extracted in the presence of a strong magnetic field threading the BH and supported by external currents flowing in the accretion disc, as shown by Blandford & Znajek (1977). The Blandford–Znajek mechanism has been widely applied to jet formation in AGN, as well as to microquasar jets and gamma ray bursts, in an attempt to match a number of observational data. Macdonald & Thorne (1982) explained this mechanism in terms of the BH membrane paradigm, in which case an imaginary stretched horizon (a conducting surface just outside of the BH event horizon) mimics the BH electrodynamics as seen by outside observers.

Different theoretical models of jet formation have been tested already by using numerical simulations. For instance, general relativistic magnetohydrodynamics (GRMHD) simulation results are consistent with models of gas pressure and magnetically driven jets (e.g., Koide et al. 1999; Mizuno et al. 2004; Nishikawa et al. 2005; Hawley & Krolik 2006), as well as with the Blandford–Znajek mechanism (e.g., Komissarov 2001; Koide 2003; McKinney & Gammie 2004).

The BH-disc magnetic connection, first mentioned by Zeldovich & Schwartzman and quoted in Thorne (1974), can occur and change the energy-angular-momentum balance of the accreted gas in the disc (e.g., Thorne et al. 1986; Blandford 1999). The first derivation of the energy and angular momentum transferred from the BH to the accretion disc by magnetic connection was developed by Li (2000a,b, 2002a). [See also Wang et al. (2003).] As the BH rotates relative to the disc, an electromotive force is generated. This drives a poloidal electric current flowing through the BH and the disc and produces an additional power on the disc. From the conservation laws of energy and angular momentum for a thin Keplerian accretion disc torqued by a BH, Li (2002a) calculated the radiation flux, the internal viscous torque, and the total power of the disc, and found that the disc can radiate even without accretion. Li (2002b) also looked for observational signatures of the magnetic connection between a BH and a disc as more energy is radiated away from the disc and showed that the magnetic connection can produce a very steep emissivity compared to the standard, thin-accretion disc model. Uzdensky (2004, 2005) obtained the numerical solution of the Grad-Shafranov equation for a magnetic connection configuration in the case of both Schwarzschild and Kerr BHs. The Grad-Shafranov equation is a non-linear, partial differential equation that describes the magnetic flux distribution of plasma in an axisymmetric system. Uzdensky found that a magnetic connection configuration can only be maintained very close to the BH. In recent years, a number of models that also include the BH-disc magnetic connection have been developed. Wang et al. (2007) proposed a toy model for the magnetic connection, in which case a poloidal magnetic field is generated by a single electric current flowing in the equatorial plane around a Kerr BH. Ma et al. (2007) derived the energy and angular momentum fluxes for a Kerr BH surrounded by an advection-dominated accretion flow disc. To solve the equations of the accretion flow, they used a pseudo-Newtonian potential. Gan et al. (2009) solved the dynamic equations for a disc-corona system and simulated its X-ray spectra by using the Monte Carlo method. Zhao et al. (2009) studied the magnetic field configuration generated by a toroidal distributed continuously in a thin accretion disc, as well as the role of magnetic reconnection in the disc to produce quasi-periodic oscillations in BH binaries. In the context of GRMHD, Koide et al. (2006) presented a 2-D GRMHD result of jet formation driven by a magnetic field produced by a current loop near a rapidly rotating BH, in which case the magnetic flux tubes connect the region between the BH ergosphere and a co-rotating accretion disc.

In this paper, we propose a model for launching relativistic jets from a geometrically thin accretion disc by means of the BH-disc magnetic connection. The model is based on the calculations of Novikov & Thorne (1973), Page & Thorne (1974), and Li (2002a), being mainly influenced by the work of Znajek (1978) and Macdonald & Thorne (1982). We use the general relativistic form of the conservation laws for the matter in a thin accretion disc to describe the disc structure when both the BH-disc magnetic connection and the jet formation are considered. Some incipient ideas which are at the base of this model were exposed in Duţan & Biermann (2005).

In Section 2, we describe the assumptions of the model. Using the general relativistic conservation laws for matter in the accretion disc (Section 4), we derive the mass flow rate into the jets (Section 3), the launching power of the jets (Section 5), and the angular momentum removed by the jets (Section 6). In Section 7, we calculate the efficiency of launching the jets and show that when the BH accretes at low rates, the spin-down of the black hole is an efficient

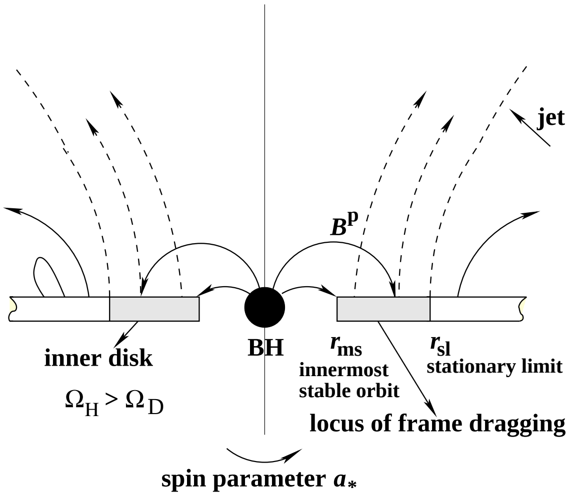

Figure 1. Schematic representation of the inner part of the accretion disc-BH-jet system. We represent the BH by the so-called stretched horizon (Macdonald & Thorne 1982), in which case the BH null horizon is replaced with a time-like physical surface endowed with electrical, mechanical, and thermodynamical properties. In the region of the inner disc, the closed magnetic field lines (solid lines) do not cross the open magnetic field lines (dashed lines); they overlap only in a line-of-sight projection. For some explanation on the structure of the magnetic field in the inner disc, the reader is referred to the text below.

mechanism to launch the jets via the accretion disc. In Section 8, we study the spin evolution of the BH and discuss conditions of BH stationary states for given mass accretion rates. In Section 9, we refer to the long lifetime of an AGN from the BH spin-down power as a particular relevance of the proposed model to the observational data. In Section 10, we present a summary of the key points, as well as our conclusions.

2 BASIC ASSUMPTIONS

• We consider that matter outside of the BH has negligible gravitational effects compared to the BH gravity and that an accretion disc settles down in the equatorial plane of a Kerr BH. The term inner disc denotes the region of the accretion disc that extends from the stationary limit surface (or ergosphere) inward to the innermost stable (circular) orbit (see Fig. 1).

• We consider the case of rapidly spinning BHs with a spin parameter $a_* \geqslant 0.95$, based on the argument by Bardeen (1973) that a strong preference for a particle to orbit in the equatorial plane requires the BH spin parameter to be close to its maximum value.

• We assume the point of view of the BH membrane paradigm (Macdonald & Thorne 1982), in which case the null horizon is replaced with a time-like physical surface endowed with electrical, mechanical, and thermodynamical properties, the so-called stretched horizon.

• We consider that closed magnetic field lines connect the BH to the inner accretion disc (e.g., Blandford 1999; Li 2000b). The poloidal component of the magnetic field transfers angular momentum and energy from the BH to the disc, thereby increasing the amount of the rest-mass energy of the infalling matter in the BH potential wall released from the accretion disc. This energy is liberated very close to the BH, where most of the accretion disc energy is available (i.e., from the inner disc), and we next assume that this energy is used to launch the jets. Because the disc energy is used to launch the jet, the inner disc beneath the jet is left radiatively inefficient. We do not exclude the possibility that the outer part of the disc, beyond the stationary limit surface, either radiates with some efficiency or drives non-relativistic winds.

• We assume that the accretion disc is geometrically thin and quasi-Keplerian. It is assumed that the particles move in both radial and vertical directions, so that the total angular momentum of the disc redistributes itself to keep a quasi-Keplerian disc. The BH magnetic torque exerted on the disc causes a removal of the angular momentum from the disc particles and a slow radial drift toward the BH. The removed disc angular momentum is carried away by the particles that form the jets. Next, we assume that the surface density of the disc and the mean radial velocity combine themselves in such a way to keep their product constant over the inner disc. This can be thought of as being an effect of the BH rotation.

• Following the calculations by Znajek (1978), see also Lovelace (1976), the BH generates a voltage difference between the horizon and a few gravitational radii of the order of $V \sim 10^{21}$ volt for a rapidly spinning BH ($a_* \sim 1$) with a mass of $10^9 M_\odot$. If we compare the electric force that acts on an electron (proton) to the gravitational pull of the BH, we get a factor of $\sim 10^{15}$ ($\sim 10^{12}$) in favour of electric force. Therefore, the disc particles can slide along the magnetic field lines (dashed lines in Fig. 1).

• To form the jet, at some point, the particles must cross the magnetic field lines. One can picture this as being due to drifts and instabilities inside the disc (Balbus & Hawley 1994, 1996, 1998). We represent the magnetic field in the inner disc similar to the magnetic field on the Sun surface. In a direct top-down view, it looks very mottled, with arcs of magnetic fields connecting different regions and other magnetic flux tubes extending into free space and allowing matter to flow out. The geometry of the magnetic field in the inner disc is not necessarily axisymmetric.

• We use the Kerr metric in cylindrical coordinates. In and near the BH equatorial plane, the metric is given by

$$ds^2 = -e^{2\nu}dt^2 + e^{2\psi}\left(d\phi - \omega dt\right)^2 + e^{2\mu}dr^2 + dz^2, \qquad (1)$$

(Page & Thorne 1974) where r, ϕ, and z are defined as the cylindrical coordinates in the asymptotic rest frame, and t is the physical time of an observer removed to infinity. The components of the metric tensor in (1) are specified by

$$e^{2\nu} = \frac{r^2\Delta}{A},\ e^{2\psi} = \frac{A}{r^2},\ e^{2\mu} = \frac{r^2}{\Delta},\ \omega = 2r_{\rm g}aA^{-1}, \qquad (2)$$

where the metric functions read

$$\Delta = r^2 - 2r_{\rm g}r + a^2,\ A = r^4 + r^2a^2 + 2r_{\rm g}ra^2, \qquad (3)$$

where G is the Newtonian gravitational constant, c is the speed of light, $a = J/Mc$ is the angular momentum of the BH about the spinning axis J per unit mass and per speed of light, and $r_{\rm g} = GM/c^2$ is the gravitational radius. The BH spin parameter is defined as $a_* \equiv J/J\max(= a/r_{\rm g})$, where $J_{\max} = GM^2/c$ is the maximal angular momentum of the BH.

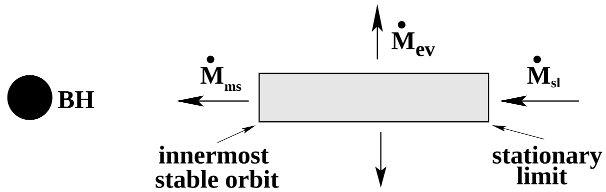

Figure 2. Schematic representation of the mass flow in the inner disc.

3 MASS FLOW RATE INTO THE JETS

We presume that the particles move in both radial and vertical directions, so that the total angular momentum of the disc redistributes itself to keep a quasi-Keplerian disc. The angular momentum transferred from the BH to the accretion disc produces a small radial infall, while the removed angular momentum is transported in the vertical direction by the particles which form the jets. The flow of (rest) mass into the jets can be described by

$$\dot{M}_{\rm jets} = \dot{M}(\rm r_{sl}) - \dot{M}(\rm r_{ms}), \quad (4)$$

where $\dot{M}(\rm r_{sl})$ and $\dot{M}(\rm r_{ms})$ denote the mass accretion rate (measured by observers at infinity) at the stationary limit surface and at the innermost stable orbit, respectively. The mass flow rate into the jets can also be expressed by

$$\dot{M}_{\rm jets} = q_{\rm jets}\dot{M}_{\rm D}, \quad (5)$$

where the parameter $q_{\rm jets}$ indicates the fraction of the inner disc mass that goes into the jets. Figure 2 shows a schematic representation of the mass flow in the inner disc.

The amount of mass that flows inward across a cylinder of radius r during a coordinate time interval Δt, when averaged by the method in Novikov & Thorne (1973), is

$$\dot{M} = -2\pi\sqrt{|g|}\,\Sigma\,\bar{v}^{\hat{r}}\,\mathcal{D}^{1/2}, \quad (6)$$

where $\sqrt{|g|} = e^{\nu+\psi+\nu} = r$ is the square root of the metric determinant (Eqs. 1 and 2), $\Sigma = 2h < \rho_0 >$ is the surface density of the disc (with h being the half thickness of the disc and $< \rho_0 >$ the density of rest mass), $\bar{v}^{\hat{r}}$ is the mass-averaged radial velocity, and $\mathcal{D} = (1 - 2/r_* + a_*^2/r_*^2)$ is one of the functions used to calculate general relativistic corrections to the Newtonian accretion disc structure. Here, $r_* = r/r_{\rm g}$ is the dimensionless radius.

Next, we estimate the mass accretion rate at one specific radius of the inner disc by using (6), so that the mass flow rate into the jets becomes

$$\dot{M}_{\rm jets} = \left[-2\pi r\,\Sigma\,\bar{v}^{\hat{r}}\,\mathcal{D}^{1/2}\right]_{\rm r_{sl}} - \left[-2\pi r\,\Sigma\,\bar{v}^{\hat{r}}\,\mathcal{D}^{1/2}\right]_{\rm r_{ms}}. \quad (7)$$

Suppose that the mass flow rate at the stationary limit surface is $\dot{m}\dot{M}_{\rm Edd}$. Then,

$$\dot{M}_{\rm jets} = \dot{m}\dot{M}_{\rm Edd}\left[1 - \frac{\dot{M}(\rm r_{ms})}{\dot{M}(\rm r_{sl})}\right]. \quad (8)$$

The product of the surface density and the mass-averaged radial velocity is proportional to the radius as $\Sigma\bar{v}^{\hat{r}} \propto r^p$. For a pure Keplerian disc, $p = -1$. To account for deviations from a pure Keplerian disc, we adopt $p = 0$; that is, the product $\Sigma\bar{v}^{\hat{r}}$ is constant for any radius of the inner disc. In this case, $\dot{M}_{\rm jets}$ at a given radius is specified by the general relativistic factor $\sqrt{|g|}\,\mathcal{D}^{1/2}$, which depends only on the BH spin parameter. This can be thought of as being an effect of the BH rotation.

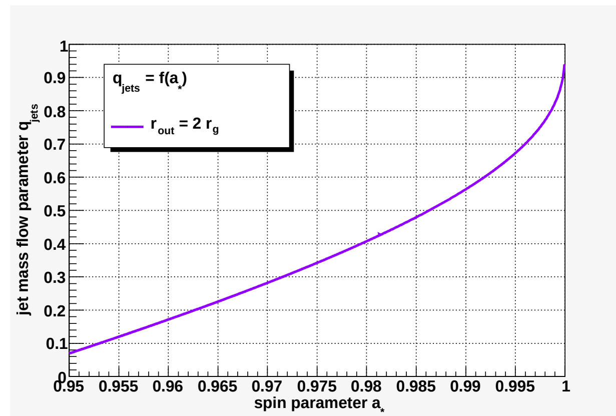

Figure 3. Jet mass flow parameter, $q_{\rm jets}$, as a function of the BH spin parameter, a_*. For $a_* \sim 1$, almost whole material of the inner disc flows into the jets ($q_{\rm jets} \simeq 0.98$); that is, the black hole stops being fed by accreting matter.

Using equations (5) and (8), as well as the expression of $\mathcal{D}$, we obtain the fraction of the mass accretion rate which can flow into the jets (i.e., the jet mass flow parameter) as

$$q_{\rm jets}(a_*) = 1 - \frac{r_{\rm ms_*}}{r_{\rm sl_*}}\left(\frac{1 - 2/r_{\rm ms_*} + a_*^2/r_{\rm ms_*}^2}{1 - 2/r_{\rm sl_*} + a_*^2/r_{\rm sl_*}^2}\right)^{1/2}, \quad (9)$$

where $r_{\rm ms_*} = r_{\rm ms}/r_{\rm g}$ and $r_{\rm sl_*} = r_{\rm sl}/r_{\rm g}$.

Figure 3 shows the jet mass flow parameter as a function of the BH spin parameter. For the extreme value of the spin parameter $a_* \sim 1$, the mass flow rate into the jets is 98 percent of the available mass flow in the inner disc. This means that in the case of extreme spin, the black hole stops being fed by accreting matter.

Suppose the inner disc would have been extended beyond the stationary limit surface. In this case, the disc particles can form the jets ($q_{\rm jets} > 0$) if and only if the BH spin parameter were $a_* > 0.755$ (plot not shown).

We mention that the results presented in this section are valid for our choice of $p = 0$, which occurs only as a consequence of the BH rotation. In the work by Donea & Biermann (1996), a model with $p = 0$ can make the distinction between the rotation curve of a Keplerian disc and the rotation curve of a sub-Keplerian jet-driven disc, the latter being shown by a reduction of the former which was mimicked with special boundary conditions.

4 ANGULAR MOMENTUM AND ENERGY CONSERVATION LAWS

The energy and angular momentum fluxes can be obtained by contracting the energy-momentum tensor of the matter in the disc with the time and axial Killing vectors of Kerr space-time. We use the angular momentum and energy conservation laws derived by Page & Thorne (1974) and include the BH-disc magnetic connection and the jet formation. To allow the mass in the disc to be accreted by the BH, the angular momentum of the disc has to be removed. We consider that the torque on the inner disc, which transports angular momentum, is produced by closed magnetic field lines that connect the BH to the accretion disc. In this case, the angular momentum of the disc is removed by the particles, which ultimately form the jets. With these, the angular momentum conservation law[1] takes the form of

$$\frac{d}{dr}\left[\left(1-q_{\rm jets}\right)\dot{M}_{\rm D}cL^{\dagger}\right]+4\pi rH=4\pi rJL^{\dagger}, \qquad (10)$$

where on the left-hand side, the first term describes the angular momentum carried by the accreting mass of the inner disc, and the second term describes the angular momentum transferred from the BH to the inner accretion disc. The term on the right describes the angular momentum carried away by the jets. $L^{\dagger}$ is the specific angular momentum of a gas particle orbiting in the accretion disc, J is the total flux of energy (of particle and magnetic origin) carried away by jets, and H is the flux of angular momentum transferred from the BH to the disc. H is defined through the magnetic torque produced by the BH on both surfaces of the accretion disc $T_{\rm HD}$ (Li 2002a)

$$T_{\rm HD}=2\int_{r_1}^{r_2}2\pi rH\,dr, \qquad (11)$$

where the limits of integration are two radii of the accretion disc with $r_1<r_2$.

Similar to the angular momentum conservation law, we can write the energy conservation law as

$$\frac{d}{dr}\left[\left(1-q_{\rm jets}\right)\dot{M}_{\rm D}c^2E^{\dagger}\right]+4\pi rH\Omega_{\rm D}=4\pi rJE^{\dagger}, \qquad (12)$$

where on the left hand-side, the first term describes the rate of the energy flow in the disc, and the second term is the magnetic torque per unit area of the disc $T_{\rm HD}\Omega_{\rm D}$ (here $\Omega_{\rm D}$ is the Keplerian angular velocity of the gas particles in the disc). The third term describes the energy flow along the jets. $E^{\dagger}$ is the specific energy of a gas particle having mass μ and orbiting in the same direction as the BH rotation (Bardeen et al. 1972):

$$E^{\dagger}\equiv\frac{E}{\mu}=\frac{r^{3/2}-2r_{\rm g}r^{1/2}+r_{\rm g}^{1/2}a}{r^{3/4}\left(r^{3/2}-3r_{\rm g}r^{1/2}+2r_{\rm g}^{1/2}a\right)^{1/2}}. \qquad (13)$$

The flux of angular momentum transferred from the BH to the accretion disc by magnetic connection has the following expression (Li 2002a):

$$H=\frac{1}{8\pi^3 r}\left(\frac{d\Psi_{\rm D}}{c\,dr}\right)^2\frac{\Omega_{\rm H}-\Omega_{\rm D}}{(-dR_{\rm H}/dr)}, \qquad (14)$$

where $\Psi_{\rm D}$ is the flux of the poloidal magnetic field lines which thread the accretion disc surface, and $\Omega_{\rm H}$ is the BH angular velocity. The derivation of Eq. (14) is based on the supposition that the accretion disc consists of a highly conducting ionised gas. This implies that (i) the accretion disc resistance is neglected in comparison with the BH surface resistance and (ii) the magnetic field lines are frozen in the accretion disc, being transported by the gas disc and rotating with $\Omega_{\rm D}$. On the other hand, the angular velocity of the magnetic field lines threading the horizon is $\Omega_{\rm H}$, due to the effect of the frame-dragging at the BH horizon. For $a_*>0.35$ and $r\geqslant r_{\rm ms}$, $\Omega_{\rm H}>\Omega_{\rm D}$, so that the BH transfers energy (and angular momentum) to the disc. For $a_*<0.35$, $\Omega_{\rm H}<\Omega_{\rm D}$, and the accretion disc transfers energy (and angular momentum) to the BH. For $a_*=0.35$, $\Omega_{\rm H}=\Omega_{\rm D}$; this condition implies that there is no energy (or angular momentum) transfer between the BH and the accretion disc by magnetic connection.

5 LAUNCHING POWER OF THE JETS

We are now in the position to calculate the launching power of the jets with the help of the conservation laws previously derived. First, we define the launching power of both jets as

$$P_{\rm jets}=2\int_{r_{\rm ms}}^{r_{\rm sl}}2\pi JE^{\dagger}r\,dr. \qquad (15)$$

Integrating the equation of the energy conservation law (Eq. 12) over the inner disc, we find the launching power of the jets:

$$P_{\rm jets}=\left(1-q_{\rm jets}\right)\dot{M}_{\rm D}c^2\left(E_{\rm sl}^{\dagger}-E_{\rm ms}^{\dagger}\right)+4\pi\int_{r_{\rm ms}}^{r_{\rm sl}}rH\Omega_{\rm D}\,dr. \qquad (16)$$

The first term describes the rest energy of the accreted matter onto the BH, and the second term describes the energy transfer from the rotating BH to the disc. $E_{\rm sl}^{\dagger}$ and $E_{\rm ms}^{\dagger}$ are the specific energy of the gas particle (Eq. 13) evaluated at the stationary limit surface and at the innermost stable orbit, respectively.

Using Eq. (14), we obtain the launching power of the jets as

$$P_{\rm jets}=\left(1-q_{\rm jets}\right)\dot{M}_{\rm D}c^2\left(E_{\rm sl}^{\dagger}-E_{\rm ms}^{\dagger}\right)+\frac{1}{2\pi^2}\int_{r_{\rm ms}}^{r_{\rm sl}}\left(\frac{d\Psi_{\rm D}}{c\,dr}\right)^2\frac{\Omega_{\rm H}-\Omega_{\rm D}}{(-dR_{\rm H}/dr)}\Omega_{\rm D}\,dr, \qquad (17)$$

where the angular velocities of the BH and the accretion disc, respectively, are

$$\Omega_{\rm H}\equiv\frac{c}{2r_{\rm g}}\frac{a_*}{1+\left(1-a_*^2\right)^{1/2}}=\frac{c}{r_{\rm g}}\Omega_{\rm H*}, \qquad (18)$$

$$\Omega_{\rm D}\equiv\frac{c}{r_{\rm g}}\frac{1}{r_*^{3/2}+a_*}=\frac{c}{r_{\rm g}}\Omega_{\rm D*}. \qquad (19)$$

To calculate the launching power of the jets, we need to evaluate both $\Psi_{\rm D}$ and $(-dR_{\rm H}/dr)$. First, we write the magnetic flux that threads the accretion disc surface,

$$\Psi_{\rm D}=\int B_{\rm D}(dS)_{z=0}, \qquad (20)$$

where $B_{\rm D}$ is the poloidal component of the magnetic field that threads the disc. The surface area between two equatorial surfaces of a Kerr BH can be calculated from

$$(dS)_{z=0}=\sqrt{\det g_{(r\phi)}}\,dr\,d\phi, \qquad (21)$$

where the determinant of the surface metric is

$$\det g_{(r\phi)}=\begin{vmatrix} g_{rr} & g_{r\phi}\\ g_{\phi r} & g_{\phi\phi}\end{vmatrix}=\begin{vmatrix} e^{2\mu} & 0\\ 0 & e^{2\psi}\end{vmatrix}=\frac{A}{\Delta}. \qquad (22)$$

This result follows from Eqs. (1), (2), and (3). With these, the surface area in Eq. (21) reads

$$(dS)_{z=0}=\left(\frac{A}{\Delta}\right)^{1/2}2\pi\,dr. \qquad (23)$$

[1] The equation describing the angular momentum conservation is derived in Appendix A.

The poloidal component of the magnetic field that threads the BH horizon $B_{\rm H}$ and the poloidal component of the magnetic field at the inner edge of the accretion disc $B_{\rm D}(r_{\rm ms})$ can be of the same order (e.g., Livio et al. 1999) and related by

$$B_{\rm H} = \zeta B_{\rm D}(r_{\rm ms})\,,\ \text{where}\ \zeta \geqslant 1. \tag{24}$$

On the other hand, the poloidal component of the magnetic field that threads the accretion disc surface scales as $B_{\rm D} \propto r^{-n}$, where $0 < n < 3$ (Blandford 1976). Consequently,

$$B_{\rm D} = B_{\rm D}(r_{\rm ms}) \left(\frac{r}{r_{\rm ms}}\right)^{-n} = \frac{B_{\rm H}}{\zeta} \left(\frac{r}{r_{\rm ms}}\right)^{-n}. \tag{25}$$

Since the BH horizon behaves, in some aspects, like a rotating conducting surface (e.g., Damour 1978; Znajek 1978; Thorne et al. 1986), it can be thought of as being a "battery" driving currents around a circuit. The energy for this comes from the BH rotation (Znajek 1978). The internal resistance of the battery in the horizon, i.e., the resistance between two magnetic surfaces that thread the horizon, is

$$dR_{\rm H} = R_{\rm H} \frac{dl}{2\pi r_{\rm H}}, \tag{26}$$

where $R_{\rm H} = 4\pi/c = 377$ ohm, dl is the horizon distance between two magnetic surfaces (see Fig. 1), $2\pi r_{\rm H}$ is the cylindrical circumference at $r = r_{\rm H}$, and $r_{\rm H} = r_{\rm g}[1+(1-a_*^2)^{1/2}] = r_{\rm g} r_{\rm H*}$ is the radius of the BH horizon (Thorne et al. 1986).

The voltage difference generated by the BH has a maximum magnitude of $V = \Omega_{\rm H}\Psi_{\rm H}$, where $\Psi_{\rm H} = B_{\rm H} A_{\rm H}$ is the magnetic flux threading the BH, and $A_{\rm H} = 8\pi r_{\rm g} r_{\rm H}$ is the surface area of the BH. Assuming that the magnetic field is carried into the BH by the accreted gas disc, we set the BH potential drop to the energy of the gas particles carried into the BH, the latter being the particle-specific energy at the innermost stable orbit. Suppose that during a first epoch, the BH accretes at a rate approximately equal to the Eddington rate[2]. This supposition provides $V^2 = \dot{M}_{\rm acc} E^{\dagger}_{\rm ms} c^2$. Therefore, the maximum value of the magnetic field that threads the BH horizon is

$$(B_{\rm H}^{\rm max})^2 = \frac{\dot{M}_{\rm Edd} c E^{\dagger}_{\rm ms,lim}}{4\pi r_{\rm g}^2 (a_{*,\rm lim})^2}, \tag{27}$$

where $a_{*,\rm lim} = 0.9982$ is the BH limiting spin in the case of a radiatively efficient accretion disc (Thorne 1974), and the corresponding specific energy at the innermost stable orbit is $E^{\dagger}_{\rm ms,lim} = 0.6759$. Although this limit of the BH spin may be even closer to its maximum value $a_* \sim 1$, it produces a negligible variation in the maximum value of the BH magnetic field. Using the expression of the gravitational radius[3],

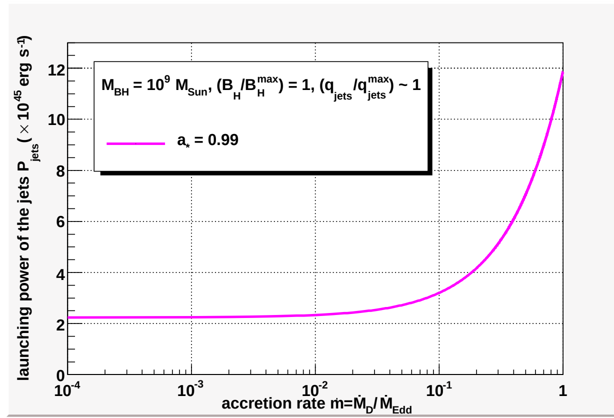

Figure 4. Launching power of the jets as a function of the mass accretion rate $\dot{m}$ (Eq. 32) for a given BH spin parameter $a_* = 0.99$. The switch from an accretion power regime to a spin-down power regime corresponds to a mass accretion rate of $\dot{m} \simeq 10^{-1.8}$.

the maximum value of the magnetic field that threads the BH horizon (Eq. 27) becomes

$$(B_{\rm H}^{\rm max})^2 = \frac{\dot{M}^{\dagger}_{\rm Edd} c E^{\dagger}_{\rm ms,lim}}{4\pi\, \varepsilon_{\rm lim} (r_{\rm g}^{\dagger})^2 (a_{*,\rm lim})^2} \left(\frac{M}{10^9 M_{\odot}}\right)^{-1}, \tag{28}$$

or

$$B_{\rm H}^{\rm max} = 0.56 \times 10^4 \left(\frac{M}{10^9 M_{\odot}}\right)^{-1/2} \text{gauss}. \tag{29}$$

The result is similar to the calculation performed by Znajek (1978). [See also Lovelace (1976).] The only difference is that we set the BH potential drop to the specific energy of the particles at the innermost stable orbit instead of the Eddington luminosity.

The continuum of the magnetic field within a narrow strip between two magnetic surfaces, which connect the BH to the accretion disc, $d\Psi_{\rm H} = d\Psi_{\rm D}$ (e.g., Wang et al. 2007), gives

$$B_{\rm H} 2\pi r_{\rm H}\, dl = -B_{\rm D} \left(\frac{A}{\Delta}\right)^{1/2} 2\pi\, dr. \tag{30}$$

Making use of Eqs. (25), (26), and (30), we obtain

$$(-dR_{\rm H}/dr) = \frac{2}{c\, r_{\rm H}^2} \frac{1}{\zeta} \left(\frac{r}{r_{\rm ms}}\right)^{-n} \left(\frac{A}{\Delta}\right)^{1/2}. \tag{31}$$

Next, we write the particle-specific energy (Eq. 13) using the dimensionless radius and the spin parameter. Then, we substitute this equation (evaluated at $r_{\rm sl_*}$ and $r_{\rm ms_*}$, respectively) together with (18) – (27), (30), and (31) for Eq. (17). So, the launching power of the jets becomes

$$\begin{aligned} P_{\rm jets} = {} & \dot{m} \dot{M}^{\dagger}_{\rm Edd} c^2 \varepsilon^{-1} (1 - q_{\rm jets}) \left(E^{\dagger}_{\rm sl_*} - E^{\dagger}_{\rm ms_*}\right) \left(\frac{M}{10^9 M_{\odot}}\right) \\ & + \dot{M}^{\dagger}_{\rm Edd} c^2\, C_* \left(\frac{B_{\rm H}}{B_{\rm H}^{\rm max}}\right)^2 \left(\frac{M}{10^9 M_{\odot}}\right) \\ & \int_{r_{\rm ms_*}}^{r_{\rm sl_*}} r_*^{1-n} {\rm R}_*^{1/2} \left(\Omega_{\rm H_*} - \Omega_{\rm D_*}\right) \Omega_{\rm D_*}\, dr_*, \end{aligned} \tag{32}$$

where

$$C_* = \frac{r_{\rm H_*}^2 r_{\rm ms_*}^n E^{\dagger}_{\rm ms_*,lim}}{4\pi\zeta(a_{*,\rm lim})^2 \varepsilon_{\rm lim}},\ {\rm R}_* = \frac{1 + a_*^2 r_*^{-2} + 2a_*^2 r_*^{-3}}{1 - 2r_*^{-1} + a_*^2 r_*^{-2}}. \tag{33}$$

[2] The Eddington accretion rate is defined from the Eddington luminosity as $\dot{M}_{\rm Edd} = L_{\rm Edd}/(\varepsilon c^2) = 4\pi G M/(\varepsilon \kappa_{\rm T} c)$, where ε is the efficiency of converting the disc energy into radiation, and $\kappa_{\rm T}$ denotes the Thomson opacity. ε depends on the BH spin parameter as $\varepsilon = 1 - E^{\dagger}_{\rm ms}$ (Thorne 1974), so that $\varepsilon = 0.06$ for a Schwarzschild BH and $\varepsilon = 0.42$ for an extremely spinning Kerr BH. We scale the BH mass to $10^9 M_{\odot}$, so that $\dot{M}_{\rm Edd} = \dot{M}^{\dagger}_{\rm Edd} \varepsilon^{-1} (M/10^9 M_{\odot})$, where $\dot{M}^{\dagger}_{\rm Edd} = 1.38 \times 10^{26}$ g s^{-1}.

[3] The gravitational radius is $r_{\rm g} = r_{\rm g}^{\dagger}(M/10^9 M_{\odot}) = 1.48 \times 10^{14} (M/10^9 M_{\odot})$ cm.

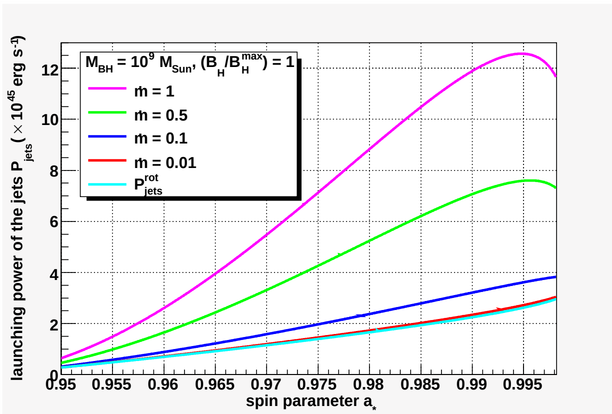

Figure 5. Launching power of the jets as a function of a_* (Eq. 32) for four values of the mass accretion rate. The mass accretion rates from top to bottom are 1, 0.5, 0.1, and 0.01. The bottom curve represents the power of the jets given by the BH spin-down $P_{\rm jets}^{\rm rot}$. Note that there is a slight difference between the two bottom curves (red and turquoise curves). In the case of very low mass accretion rates, $\dot{m} < 0.01$, $P_{\rm jets}$ is approximately equal to the BH spin-down power.

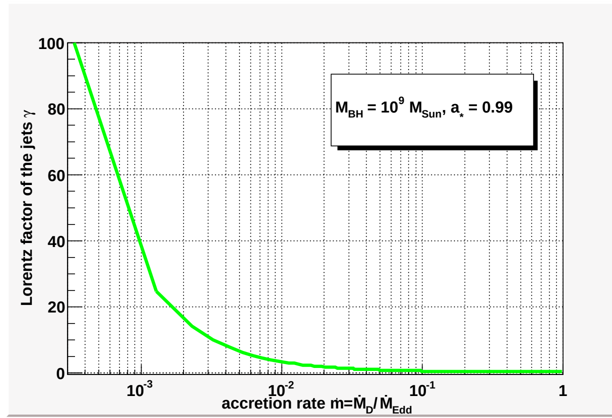

Figure 6. Lorentz factor of the jets as a function of $\dot{m}$ (Eq. 35). The jets are launched with a bulk Lorentz factor $\gamma > 2$ when the mass accretion rate $\dot{m} < 10^{-1.8}$, which corresponds to the spin-down power regime. In the case of the accretion power regime, $\dot{m} > 10^{-1.8}$, the jets are sub-relativistic, $\gamma < 2$.

For the following calculations, we consider the strength of the magnetic field in Eq. (32) to be as high as its maximum value $B_H \sim B_{\rm H}^{\rm max}$. On the right-hand side, the first term represents the accretion power of the inner disc, and the second term represents the BH spin-down power transferred to the inner disc by magnetic connection. So, Eq. (32) can also read:

$$P_{\rm jets} = P_{\rm jets}^{\rm acc} + P_{\rm jets}^{\rm rot}. \qquad (34)$$

In Fig. 4, we plot the launching power of the jets as a function of the mass accretion rate. The plot shows that a transition from an accretion power regime to a spin-down power regime is produced for $\dot{m} \simeq 10^{-1.8}$. So, we have: (1) an accretion power regime in which case $\dot{m} > 10^{-1.8}$ and the dominating term in the launching power of the jets is $P_{\rm jets}^{\rm acc}$, and (2) a spin-down power regime in which case $\dot{m} < 10^{-1.8}$ and the dominating term in the launching power of the jets is $P_{\rm jets}^{\rm rot}$.

In Eq. (32), the launching power of the jets depends on: (i) the mass accretion rate $\dot{m}$, (ii) the BH mass M, (iii) the BH spin parameter a_*, (iv) the power-law index n, and (v) the ratio of the magnetic field strengths ζ. We chose the last two parameters as follows: the power-law index n is taken to be 2 for a frozen magnetic field (Alfvén 1963), and ζ is set by taking its value corresponding to the maximum of the launching power of the jet, which is one. Therefore, for the following calculations, we consider $n = 2$ and $\zeta = 1$.

In Fig. 5, we plot the launching power of the jets as a function of the BH spin parameter, for a BH mass of $10^9 M_\odot$, given four values of the mass accretion rate ($\dot{m} = 1, 0.5, 0.1,$ and 0.01), as well as the BH spin-down power contribution to the jets power (bottom curve). Because the area of the inner disc from where the jets are launched increases with an increase of a_*, there is a dominating trend of the jet power to increase as well, except for a_* close to the extreme value. The turn-over of the curve is produced due to the general relativistic factor that appears in the term $(1 - q_{\rm jets})$ of the accretion power. In the case of the spin-down power regime, the jet power is $\sim 10^{45}$ erg s^{-1}, which is only 10^{-2} of the Eddington luminosity of a 10^9 solar mass BH. This value of $P_{\rm jets}^{\rm rot}$ is comparable to the maximum rate of energy extraction by the Blandford–Znajek mechanism, which is $\sim 10^{45}$ erg s^{-1} for a BH mass of $10^9 M_\odot$ and a_* close to the extreme value [Eq. 4.50 of Thorne et al. (1986)]. For a lower mass of the BH, the jet power decreases, as the launching power of the jets is proportional to the BH mass.

The bulk Lorentz factor of the jets γ, defined by

$$P_{\rm jets} = \gamma \dot{M}_{\rm jets} c^2 = \gamma q_{\rm jets} \dot{m} \dot{M}_{\rm Edd} c^2, \qquad (35)$$

is drawn in Fig. 6 as a function of the mass accretion rate. The jets are launched with a relativistic speed of $0.9 - 0.995\, c$ (or $\gamma = 2 - 10$, which is the typical bulk Lorentz factor for an AGN jet) when the mass accretion rate $\dot{m} \in [10^{-2.5}, 10^{-1.8}]$, i.e., these jets correspond to the spin-down power regime. In the case of the accretion power regime, the jets are sub-relativistic ($\gamma < 2$). Our approach is simple in the sense that the jet has constant properties across its entire cross-section. There is no significant variation of γ with the BH spin parameter ($a_* > 0.95$) for a given mass accretion rate.

6 RATE OF THE DISC ANGULAR MOMENTUM REMOVED BY THE JETS

Now, we define the rate of the disc angular momentum removed by the jets as

$$J_{\rm jets} = 2 \int_{r_{\rm ms}}^{r_{\rm sl}} 2\pi J L^{\dagger} r dr. \qquad (36)$$

Using the angular momentum conservation law (Eq. 10), the rate of the disc angular momentum removed by the jets can be written as

$$J_{\rm jets} = (1 - q_{\rm jets})\, \dot{M}_{\rm D} c \left(L_{\rm sl}^{\dagger} - L_{\rm ms}^{\dagger} \right) + 4\pi \int_{r_{\rm ms}}^{r_{\rm sl}} r H dr, \quad (37)$$

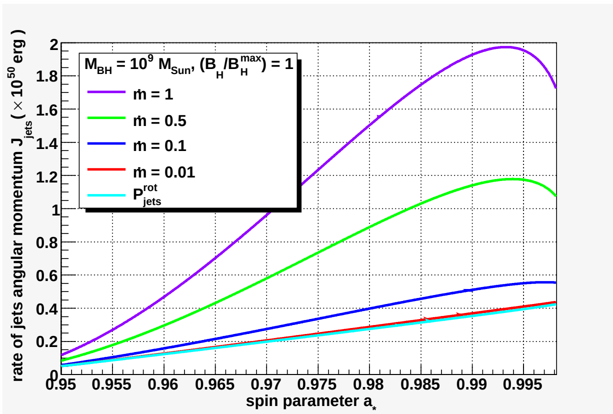

Figure 7. Rate of the disc angular momentum removed by the jets (Eq. 39) as a function of the BH spin parameter a_* for four values of the mass accretion rate. The mass accretion rates from top to bottom are 1, 0.5, 0.1, and 0.01. The bottom curve represents the BH spin-down transferred to the inner accretion disc by magnetic connection. Note that there is a slight difference between the two bottom curves (red and turquoise curves).

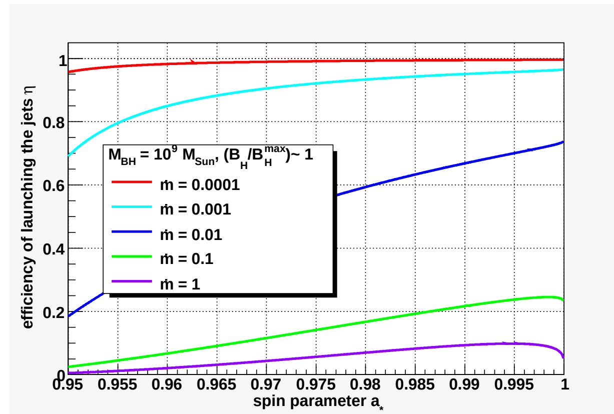

Figure 8. Efficiency of jet launching (Eq. 41) as a function of the BH spin parameter a_* for different mass accretion rates. The mass accretion rates from the top to bottom are 0.0001, 0.001, 0.01, 0.01, and 1. For very low mass accretion rates, $\dot{m} < 0.001$, the BH spin-down becomes a very efficient mechanism of launching the jets via the accretion disc.

where the dimensionless specific angular momentum of the gas particle orbiting in the accretion disc is

$$L^{\dagger}(r_*) = r_*^{-1/2} \frac{1 - 2a_* r_*^{-3/2} + a_*^2 r_*^{-2}}{\left(1 - 3r_*^{-1} + 2a_* r_*^{-3/2}\right)^{1/2}}. \qquad (38)$$

Combining Eq. (37) with (18) – (27), (30), and (31), the rate of the disc angular momentum removed by the jets becomes

$$J_{\rm jets} = \dot{m}\dot{M}_{\rm Edd}^{\dagger} c\, r_{\rm g} \varepsilon^{-1} (1 - q_{\rm jets}) \left(L_{\rm sl_*}^{\dagger} - L_{\rm ms_*}^{\dagger}\right) \left(\frac{M}{10^9 M_{\odot}}\right) + \dot{M}_{\rm Edd}^{\dagger} c\, r_{\rm g} {\rm C}_* \left(\frac{B_{\rm H}}{B_{\rm H}^{\rm max}}\right)^2 \left(\frac{M}{10^9 M_{\odot}}\right) \int_{r_{\rm ms_*}}^{r_{\rm sl_*}} r_*^{1-n} {\rm R}_*^{1/2} \left(\Omega_{\rm H_*} - \Omega_{\rm D_*}\right) dr_*, \qquad (39)$$

where C_* and R_* are defined by Eq. (33). $L_{\rm sl_*}^{\dagger}$ and $L_{\rm ms_*}^{\dagger}$ are the specific angular momentum of the gas particle (Eq. 38) evaluated at the stationary limit surface and at the innermost stable orbit, respectively. We can also write the disc angular momentum removed by the jets as the sum of two components, the accretion and the rotation parts,

$$J_{\rm jets} = J_{\rm jets}^{\rm acc} + J_{\rm jets}^{\rm rot}. \qquad (40)$$

Figure 7 shows the rate of the disc angular momentum removed by the jets as a function of the spin parameter of the BH, given four values of the mass accretion rate ($\dot{m} = 1$, 0.5, 0.1, and 0.01), as well as the BH spin-down $J_{\rm jets}^{\rm rot}$ (bottom curve). To know how this angular momentum is transported by the jets, further models must be employed.

7 EFFICIENCY OF JET LAUNCHING

We define the efficiency of jet launching as the ratio of the launching power of the jets to the total power that comes from the rest-mass energy of the infalling matter in the BH potential wall and from the rotational energy of the BH. Thus,

$$\eta = \frac{P_{\rm jets}}{\dot{m}\dot{M}_{\rm Edd} c^2 + P_{\rm jets}^{\rm rot}}. \qquad (41)$$

In Fig. 8, we plot the efficiency of jet launching for the range of the mass accretion rate $\dot{m} \in [0.0001, 1]$. For very low mass accretion rates, $\dot{m} < 0.001$, the efficiency of jet launching reaches values close to unity, in which case the spin-down of the black hole becomes a very efficient mechanism to launch the jets via the accretion disc.

8 SPIN EVOLUTION OF THE BLACK HOLE

Theoretically, a Kerr BH can be spun up to a state with a spin parameter whose maximum value is $a_* = 1$. As the spin evolves, a Kerr BH can achieve a stationary state. A theorem established by Hawking (1972) states that a BH is in a stationary state if and only if the BH is either static or axisymmetric. Suppose we have a Kerr BH. Perturbating fields can, however, deflect the spin orientation away from the symmetry axis. In this case, the BH must either spin down until a static (Schwarzschild) BH is reached or evolve in such a way that it aligns its spin with the perturbative field orientation.

Next, we study the BH spin evolution and seek the maximum spin parameter that corresponds to a stationary state of the BH, when both the BH-disc magnetic connection and the jet formation are considered. Thorne (1974) calculated the influence of photon capture on the spin evolution of the BH and found a limiting state of $a_{*\rm lim} \simeq 0.9982$. This limit does not apply to our model since the inner disc is not radiative, as in the case of Thorne's model. Instead, it drives the jets. We consider this limit only to determine the maximum value of the BH magnetic field, given at the time when the BH accretes at near the Eddington limit.

Bardeen (1970) showed that the mass and angular momentum of the BH can be changed by the specific energy and angular momentum of the particles carried into the BH. The

BH mass (and the angular momentum) variation equals the value of the specific energy (and the angular momentum) at the innermost stable orbit multiplied by the rest mass accreted (dM_0) if no other stress energy is allowed to cross the horizon. That is,

$$dM = E^{\dagger}_{\rm ms}\, dM_0 \;\; \text{and} \;\; dJ = J^{\dagger}_{\rm ms}\, dM_0, \qquad (42)$$

where $E^{\dagger}_{\rm ms}$ and $J^{\dagger}_{\rm ms}$ are the specific energy and angular momentum of the particles evaluated at the innermost stable orbit. Using Eq. (42), one can obtain the differential equation that describes the spin evolution of the BH due to matter accretion:

$$\left(\frac{da_*}{d\ln M}\right)_{\rm matter} = \frac{c}{GM}\left(\frac{dJ}{dM}\right) - 2a_*. \qquad (43)$$

Now, we consider the magnetic extraction of the BH rotational energy through the BH-disc magnetic connection. The spin evolution law (Eq. 43) will be changed due to the counter-acting torque exerted on the BH by the magnetic field that connects the BH to the accretion disc. The energy and angular momentum lost (or gained, depending on the angular velocities of the BH and disc, Eq. 14) by the BH through the BH-disc magnetic connection are (see, e.g., Li 2002a):

$$c^2\left(\frac{dM}{dt}\right)_{\rm HD} = 2P_{\rm HD} \;\; \text{and} \;\; \left(\frac{dJ}{dt}\right)_{\rm HD} = 2T_{\rm HD}, \qquad (44)$$

where $P_{\rm HD} = \Omega_{\rm H} T_{\rm HD}$, and the factor '2' comes from the fact that the accretion disc has two surfaces. Adding the effects of the BH spin-up by accretion (Eq. 42) and the BH spin-down by magnetic connection (Eq. 44), the equations for evolution of the BH mass and the BH angular momentum become:

$$c^2\left(\frac{dM}{dt}\right) = (1-q_{\rm jets})\,\dot{M}_{\rm D} c^2 E^{\dagger}_{\rm ms} + c^2\left(\frac{dM}{dt}\right)_{\rm HD}, \qquad (45)$$

$$\left(\frac{dJ}{dt}\right) = (1-q_{\rm jets})\,\dot{M}_{\rm D} L^{\dagger}_{\rm ms} + \left(\frac{dJ}{dt}\right)_{\rm HD}. \qquad (46)$$

Using these two equations, we can express the BH spin evolution as

$$\left(\frac{da_*}{d\ln M}\right)_{\rm total} = \frac{c}{GM}\frac{(1-q_{\rm jets})\dot{M}_{\rm D}L^{\dagger}_{\rm ms} + \left(\frac{dJ}{dt}\right)_{\rm HD}}{(1-q_{\rm jets})\dot{M}_{\rm D}E^{\dagger}_{\rm ms} + \left(\frac{dM}{dt}\right)_{\rm HD}} - 2a_*, \qquad (47)$$

when both the BH spin-up by accreting matter and the BH spin-down due to the angular momentum transferred from the BH to the accretion disc are considered. From Eqs. (43) and (47), the spin-down of the BH by means of BH-disc magnetic connection is described by

$$\left(\frac{da_*}{d\ln M}\right)_{\rm HD} = \left(\frac{da_*}{d\ln M}\right)_{\rm total} - \left(\frac{da_*}{d\ln M}\right)_{\rm matter}. \qquad (48)$$

Figure 9 shows the spin evolution of a Kerr BH for two values of the mass accretion rate $\dot{m} = 0.1$ and $\dot{m} = 0.01$, respectively. The purple line is the driving torque by which the matter spins up the BH [Eq. 43; see also fig. 6 in Thorne (1974)], and the blue and green lines represent the counter-acting torque on the BH due to transfer of rotational energy from the BH to the disc (Eq. 48) for $\dot{m} = 0.1$ and $\dot{m} = 0.01$, respectively. The crossing point of the plots corresponds to the spin parameter for which the BH is in a stationary state.

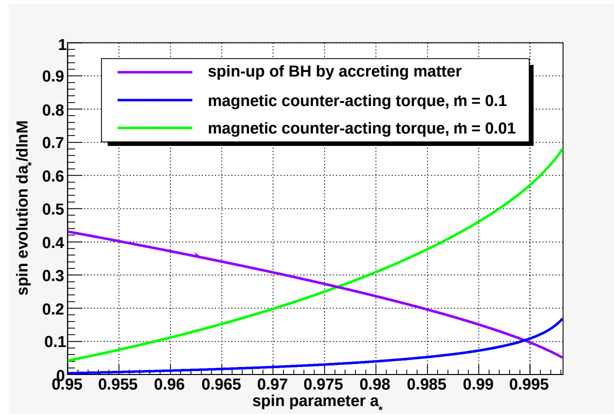

Figure 9. Spin evolution of a Kerr BH. The purple line is the driving torque by which the matter spins up the BH (Eq. 43). The blue and green lines represent the counter-acting torque on the BH due to transfer of rotational energy from the BH to the disc (Eq. 48) for $\dot{m} = 0.1$ and $\dot{m} = 0.01$, respectively. For these two cases of the mass accretion rate, the stationary state of the BH corresponds to $a_* \simeq 0.9944$ and $a_* \simeq 0.9762$, respectively.

For $\dot{m} = 0.1$, if the BH initially rotates with $a_* = 0.9982$, the BH may spin down to a stationary state with a maximum spin parameter of $a_* = 0.9944$. For $\dot{m} = 0.01$, the maximum spin parameter is $a_* = 0.9762$, whereas for $\dot{m} = 0.001$, the maximum spin is $a_* = 0.9525$. Thus, as the mass accretion rate decreases, the maximum spin parameter corresponding to a stationary BH decreases as well. On the other hand, as the mass accretion rate decreases, the magnetic torque reaches values close to unity, so it is greater than 0.43, the maximum value of the matter torque. This implies deviations from pure Keplerian orbits, and so the possibility to drive away the excess angular momentum of the disc in the form of jets when the BH spin-down power is considered. A further analysis of the spin evolution (which is not explicitly shown in Fig. 9) suggests that a BH needs a mass accretion rate of at least $\dot{m} \sim 0.001$ for its spin to stay high ($a_* \geqslant 0.95$). For lower mass accretion rates $\dot{m} < 0.001$, the BH may spin down continuously until the BH reaches a static state. It can spin-up again to $a_* \geqslant 0.95$ if a large amount of matter is provided by accretion (or by merging, which is not discussed here). In this case, the amount of accreted mass should be a factor of about 1.84 from the initial mass of the BH (Thorne 1974).

9 MAXIMUM LIFETIME OF THE AGN FROM THE BH SPIN-DOWN POWER

In this section, we calculate the time-scale needed for a Kerr BH to spin down from $a_* \sim 1$ to 0.95, which can then be related to the maximum lifetime of the AGN, provided that the BH was spun up to nearly its maximum spin during a phase when the AGN was active. The AGN can be active as long as the rest-mass energy of the infalling matter in the BH potential wall is converted into observed radiation energy. At low mass accretion rates, the energy output is dominated by the energy in the jets rather than the disc luminosity. This can be the case of an AGN that was active during an earlier phase in its evolution. Therefore, such an

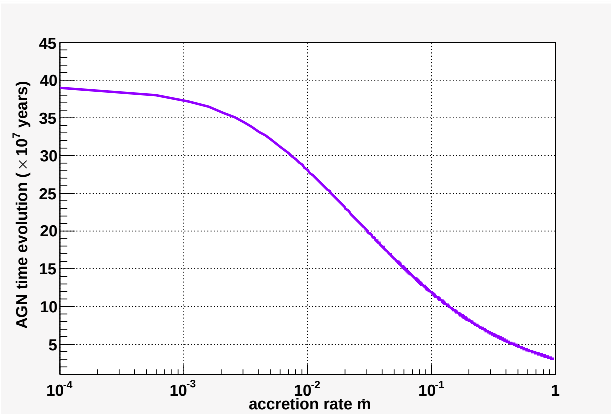

Figure 10. Lifetime of the AGN from the BH spin-down power as a function of the mass accretion rate. The lifetime of the AGN is $\sim 3 \times 10^7$ yr when using the accretion power. The BH spin-down power adds to the lifetime of the AGN, for instance, 2.8×10^8 yr, 3.9×10^8 yr, and $\sim 4 \times 10^8$ yr when $\dot{m} = 10^{-2}$, $\dot{m} = 10^{-3}$, and $\dot{m} < 10^{-4}$, respectively, while the BH spin decreases from $a_* \sim 1$ to 0.95.

AGN can have a longer lifetime through the additional use of its BH spin-down power, despite having a very-low mass-accretion rate ($\dot{m} < 0.1$). Following the well-known work by Salpeter (1964), the time needed to fuel the AGN to a bolometric luminosity $L_{\rm bol} \sim 10^{45}$ erg s^{-1} can be $\sim 10^7$ yr for a typical radiative efficiency of $\varepsilon = 0.1$. Moreover, the lifetime of high accreting AGN (and quasars) was constrained by recent observations to the range $\sim 10^7 - 10^8$ yr (e.g., Porciani et al. 2004; Hopkins & Hernquist 2009).

Next, we estimate the maximum lifetime of the AGN. Differentiating the BH angular momentum $J = Mca = (GM^2/c)\, a_*$ with respect to time t, the BH time evolution is specified by

$$\left(\frac{da_*}{dt}\right) = \frac{c}{GM^2}\left(\frac{dJ}{dt}\right) - 2\frac{a_*}{M}\left(\frac{dM}{dt}\right). \qquad (49)$$

Integrating this equation, the time interval over which the BH spin evolves between two given values of a_* is

$$t = \int_{a_{*1}}^{a_{*2}} \left[\frac{c}{GM^2}\left(\frac{dJ}{dt}\right) - 2\frac{a_*}{M}\left(\frac{dM}{dt}\right)\right]^{-1} da_*, \qquad (50)$$

where (dJ/dt) can be obtained from Eqs. (46) and (48), and (dM/dt) from Eqs. (45) and (48). With the above equation, we can estimate the lifetime of the AGN. The time interval is not dependent on the BH mass.

In Fig. 10, we plot the time evolution of the AGN as a function of the mass accretion rate (Eq. 50), when the BH spin parameter decreases from $a_* \sim 1$ to 0.95. For a mass accretion rate close to the Eddington limit, the lifetime of the AGN is about 3×10^7 yr. The lifetime curve moves toward lower mass accretion rates for another $\sim 10^8$ yr, when the AGN uses its BH spin-down power to launch the jets, so that the total lifetime of the AGN can be much longer than the Hubble time, $t_{\rm H} \sim 10^{10.14}$ yr. The maximum lifetime of the AGN is, however, dependent on the mass accretion rate. The maximum lifetime of the AGN from the BH spin-down power is, for instance, $\sim 2.8 \times 10^8$ yr, $\sim 3.9 \times 10^8$ yr, $\sim 4 \times 10^8$ yr for $\dot{m} = 10^{-2}$, $\dot{m} = 10^{-3}$, and $\dot{m} < 10^{-4}$, respectively. In the latter case, the BH may not attain a stationary state and spins down until a static BH is reached. The lifetime of the AGN scales with the strength of the BH magnetic field relative to its maximum value as $t \sim (B_{\rm H}/B_{\rm H}^{\rm max})^{-2}$. Therefore, if the $B_{\rm H}$ is a factor of k lower than $B_{\rm H}^{\rm max}$, the maximum lifetime of the AGN will be a factor of k^2 larger. For instance, when $k = 7$ and $\dot{m} = 10^{-2}$, one obtains the exact Hubble time.

Now, we compare our results to the lifetime of an AGN power by the Blandford–Znajek mechanism. The total energy that can be extracted by the Blandford–Znajek mechanism is (e.g., Li 2000a):

$$E_{\rm BZ} \simeq 0.09Mc^2 \simeq 1.6 \times 10^{62}\ {\rm erg}\left(\frac{M}{10^9 M_\odot}\right). \qquad (51)$$

The maximum rate of energy extraction by the Blandford–Znajek mechanism is (Eq. 4.50 of Thorne et al. (1986)):

$$P_{\rm BZ} \simeq 10^{45}\ {\rm ergs}^{-1}\left(\frac{B_{\rm H}}{10^4 {\rm G}}\right)^2\left(\frac{M}{10^9 M_\odot}\right)^2, \qquad (52)$$

for a BH with a_* close to the extreme value. Therefore, the lifetime of an AGN powered by the Blandford–Znajek mechanism is:

$$t_{\rm BZ} = \frac{E_{\rm BZ}}{P_{\rm BZ}} \simeq 5 \times 10^9\ {\rm yr}\left(\frac{B_{\rm H}}{10^4 {\rm G}}\right)^{-2}\left(\frac{M}{10^9 M_\odot}\right)^{-1}, \qquad (53)$$

which is not dependent of the BH mass, as $B_{\rm H}$ scales with $(M/10^9 M_\odot)^{-1/2}$. Except for the exact time scale, our result (Eq. 50) scales with the magnetic field and with the BH mass exactly the same way as for the Blandford-Znajek mechanism.

Moving back to our results, if the mass accretion rate changes over the whole life of the AGN from $\dot{m} \sim 10^{-1.8}$ to $\dot{m} < 10^{-4}$, the maximum lifetime of the AGN can be even longer than that of an AGN powered by the Blandford–Znajek mechanism. In summary, the maximum lifetime of the AGN can be much longer than $\sim 10^7$ yr when using the BH spin-down power. The lifetime is dependent on the mass accretion rate, as well as on the factor $(B_{\rm H}/B_{\rm H}^{\rm max})$. We mention that the results presented here refer only to rapidly spinning BHs ($a_* \geqslant 0.95$).

10 SUMMARY AND CONCLUSIONS

Starting from the general relativistic conservation laws for matter in a thin accretion disc, we included both the BH-disc magnetic connection and the jet formation. For this situation, we derived the mass flow rate into the jets, the launching power of the jets, the angular momentum removed by the jets, the efficiency of launching the jets, the maximum spin parameter attained by a stationary BH, and the maximum lifetime of an AGN.

- We found that the mass flow rate into the jets is dependent on the BH spin parameter. For the extreme value of the spin parameter $a_* \sim 1$, the mass flow rate into the jets is 98 percent of the available mass flow in the inner disc. This means that in the case of extreme spin, the black hole stops being fed by accreting matter. As a possible alternative, the jets may have no matter right at the beginning (i.e., they are Poynting flux jets) and get it only very quickly from drifts just above the disc or the surrounding wind (i.e., indirectly from the disc). This may only be relevant for extremely low accretion rates.

• In this work, we considered the case of rapidly spinning BHs with a spin parameter of $a_* \geqslant 0.95$ and a mass of $10^9 M_\odot$, and we assumed that the whole inner disc power is used to drive the jet. We found that at low mass accretion rates, the jet power can be supplied by the BH rotational energy via the inner accretion disc, which is located inside the BH ergosphere. The switch from an accretion power regime to a spin-down power regime corresponds to a mass accretion rate of $\dot{m} \simeq 10^{-1.8}$. In the case of the spin-down power regime ($\dot{m} < 10^{-1.8}$), the jet power is $\sim 10^{45}$ erg s^{-1}, which is only 10^{-2} of the Eddington luminosity of a 10^9 solar mass BH. This is comparable to the maximum rate of energy extraction by the Blandford–Znajek mechanism, which is $\sim 10^{45}$ erg s^{-1} for a BH mass of $10^9 M_\odot$ and a_* close to the extreme value. The jets are launched with a relativistic speed of $0.9 - 0.995\,c$ (or $\gamma = 2 - 10$, which is the typical bulk Lorentz factor for an AGN jet) when the mass accretion rate $\dot{m} \in [10^{-2.5}, 10^{-1.8}]$. In the case of the accretion power regime, the jets are sub-relativistic, in the simple approach used here, with constant properties across the entire cross-section of the jet. The jets remove the disc angular momentum at a rate which is dependent on the BH spin parameter. To know how this angular momentum is transported by the jets, further models have to be employed. The efficiency of jet launching is higher at low mass accretion rates, reaching values close to unity for $\dot{m} \sim 10^{-4}$.

• Considering the balance between the BH spin-up by accreting matter and the BH spin-down due to the magnetic counter-acting torque on the BH, we determined the maximum spin parameter which corresponds to the BH stationary state. The maximum spin value shifts towards $a_* = 0.95$ as the mass accretion rate decreases. For instance, the maximum spin parameter corresponding to $\dot{m} = 0.1$, $\dot{m} = 0.01$, and $\dot{m} = 0.001$ is $a_* = 0.9944$, $a_* = 0.9762$, and $a_* = 0.9525$, respectively. At lower mass accretion rates ($\dot{m} < 0.001$), the BH undergoes a spin-down process towards a static BH. The BH never reaches a stationary state unless a large amount of matter is provided (perhaps by star capture or by merging) to spin up the BH again to $a_* \geqslant 0.95$.

• We showed that an AGN can have a much longer lifetime than $\sim 10^7$ yr when using the BH spin-down power, and the maximum lifetime is dependent on the mass accretion rate, as well as on the factor $(B_{\rm H}/B_{\rm H}^{\max})$. After an accretion-dominated phase of about 3×10^7 yr, the AGN can live off of the BH spin-down power for another 10^8 yr. The BH spin-down power adds to the lifetime of the AGN, for instance, $\sim 2.8 \times 10^8$ yr, $\sim 3.9 \times 10^8$ yr, $\sim 4 \times 10^8$ yr for $\dot{m} = 10^{-2}$, $\dot{m} = 10^{-3}$, and $\dot{m} < 10^{-4}$, respectively. Moreover, if the $B_{\rm H}$ is a factor of k lower that $B_{\rm H}^{\max}$, then the lifetime of the AGN will be a factor of k^2 larger. For $k = 7$ and $\dot{m} = 10^{-2}$, one obtains the exact Hubble time. Another possibility is that the mass accretion rate changes over the whole life of the AGN from $\dot{m} \sim 10^{-1.8}$ to $\dot{m} < 10^{-4}$. In this case, the maximum lifetime of the AGN can be even longer than that of an AGN powered by the Blandford–Znajek mechanism, which is $\sim 5 \times 10^9$ yr for a BH with a_* close to the extreme value. However, it will be difficult to predict a maximum lifetime of the AGN for this possibility, since there is no mechanism to control the change of the mass accretion rate over long intervals of time.

A study of the BH spin-down regime in conjunction with the observed radio flux density from flat-spectrum core sources will be discussed somewhere else. The results presented in this paper are dependent on our assumptions that a BH-disc magnetic connection exists. Closed magnetic field lines in the BH ergosphere may be produced by a current ring in the vicinity of the BH. Models for the magnetic connection where a poloidal magnetic field is generated by a single electric current flowing in the BH equatorial plane or at the inner edge of the accretion disc were proposed, for instance, by Li (2002c) and Wang et al. (2007). The key parameters of our model are, however, the BH mass, the BH spin, and the mass accretion rate. Although a numerical simulation of jet formation from the ergosphere of a rapidly spinning BH with a BH-disc magnetic connection has not been performed yet, this can be one of the challenges to be faced by numerical relativists.

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APPENDIX A: DERIVATION OF EQUATION 4

Here, we present the derivation of the angular momentum conservation law for the matter that flows in the accretion disc (Eq. 10). This derivation is based on the general-relativistic angular-momentum conservation law that describes the structure of a geometrically thin accretion disc (Page & Thorne 1974). Based on this conservation law, Li (2002) derived the conservation law that includes the BH-disc magnetic connection, and Donea & Biermann (1996) derived the conservation law that includes the jet. In a slightly different manner, our result is obtained when both the BH-disc magnetic connection and the jet formation are considered. The angular-momentum conservation law can be obtained by contracting the energy-momentum tensor of the matter in the disc with the axial Killing vector of Kerr spacetime. This conservation law can be simplified to a useful form as in eq. (23) of Page & Thorne (1974), where the differential form of the conservation law is converted to an integral form by integrating this differential form over the 3-volume of the disc between the radii r and $r+\Delta r$. Gauss's theorem is then applied to convert the volume integral to a surface integral. For details on this procedure and the meaning of the notations used, the reader is referred to the paper by Page & Thorne (1974). Below, we consider the first and second integrals of their equation, keeping the non-vanishing terms:

$$\begin{aligned} &\left\{ \int_{-h}^{h} (2\pi\Delta t) \left(<\rho_0> u_\phi u^r + <t^r_\phi> \right) \sqrt{|g|} dz \right\}_r^{r+\Delta r} \\ &= \left\{ (2\pi\,\Delta t)(\Sigma L^\dagger u^r + W^r_\phi)\sqrt{|g|} \right\}_r^{r+\Delta r} \\ &= \Delta t\,\Delta r \left[-(\dot{M}_{\rm D} - \dot{M}_{\rm jets}) L^\dagger + 2\pi r W^r_\phi \right]_{,r}, \end{aligned} \tag{A1}$$

where we made use of the rest-mass conservation law that includes the mass flow into the jets , $\dot{M}_{\rm D} - \dot{M}_{\rm jets} = -2\pi r \Sigma u^r$, and

$$\begin{aligned} &\left\{ \int_{r}^{r+\Delta r} (2\pi\Delta t) \left(<\rho_0> u_\phi u^z + u_\phi <q^z> \right) \sqrt{|g|} dr \right\}_{-h}^{h} \\ &= 2\,(2\pi\,\Delta t)(\Sigma u^z + F) L^\dagger \sqrt{|g|}\Delta r = 2\,(2\pi r\,\Delta t\,\Delta r) J L^\dagger, \end{aligned} \tag{A2}$$

where $J = \Sigma u^z + F$ denotes the total flux of energy (of particle and magnetic origin) transported by jets.

Adding the two integrals, eq. (23) of Page & Thorne becomes:

$$\left[(\dot{M}_{\rm D} - \dot{M}_{\rm jets}) L^\dagger - 2\pi r W^r_\phi \right]_{,r} = 4\pi J L^\dagger. \tag{A3}$$

Now, using the definition of the magnetic torque produced by the BH on both surfaces of the accretion disc which is given by Li (see Eqs. 11 and 14) and including c, Eq. A3 becomes:

$$\frac{d}{dr} \left[(1 - q_{\rm jets})\, \dot{M}_{\rm D} c L^\dagger \right] + 4\pi r H = 4\pi r J L^\dagger, \tag{A4}$$

when both the BH-disc magnetic connection and the jet formation are considered.